On the way to M-theory

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Outline

- Generalized Buscher approach to T-duality
- Non-geometry
- Unification of all T-dual theories in double space

Consistent superstring theories

- There are five consistent versions of superstring theory
  type I, type IIA, type IIB, and two heterotic string theories
  Mystery: why there is not just one consistent formulation?
- These theories are related in nontrivial ways
  Witten's conjecture: They are just special limiting cases of an eleven-dimensional theory called M-theory
- Different string theories related by T-duality and S-duality
  - T-duality: Strings propagating on completely different spacetime geometries may be physically equivalent
  - S-duality: System of strongly interacting strings can be viewed as a system of weakly interacting strings

- In absence of an understanding structure of M-theory, Witten has suggested that the M should stand for
  Magic, Mystery, or Membrane
- How to find fundamental formulation of M-theory?
  Construct the theory that contains initial and all dual theories
- We will investigate only T-duality

Action

Closed string action

$$S[x] = \kappa \int_{\Sigma} d^2\xi \sqrt{-g} \left[\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}[x] + \frac{\epsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}[x] \right] \partial_\alpha x^\mu \partial_\beta x^\nu$$

Action principle $\delta S = 0$ gives equations of motion and boundary conditions

$$\gamma_\mu^{(0)}(x) \delta x^\mu /_{\sigma=\pi} \quad - \quad \gamma_\mu^{(0)}(x) \delta x^\mu /_{\sigma=0} = 0$$

where we define σ-momentum

$$\gamma_\mu^{(0)}(x) \equiv \frac{\delta S}{\delta x'^\mu} = \kappa \Big(2 B_{\mu\nu} \dot{x}^\nu - G_{\mu\nu} x'^\nu \Big)$$

Buscher T-duality procedure for constant background 1

- Buscher procedure:
  - gauging global symmetries $\delta x^\mu = \lambda^\mu$
    $\partial_\alpha x^\mu \to D_\alpha x^\mu = \partial_\alpha x^\mu + v_\alpha^\mu$,
    - v_α^μ gauge field
    - D_α covariant derivative
  - Field strength $F_{\alpha\beta}^\mu = \partial_\alpha v_\beta^\mu - \partial_\beta v_\alpha^\mu$
  - T-dual theory must be Physically equivalent to initial theory
    $F_{01}^\mu \equiv F^\mu = 0$

Buscher T-duality procedure for constant background 2

- Invariant Action

$$S_{inv}(x,y,v) = \kappa \int_\Sigma d^2\xi \left[(\frac{\eta^{\alpha\beta}}{2} G_{\mu\nu} + \varepsilon^{\alpha\beta} B_{\mu\nu}) D_\alpha x^\mu D_\beta x^\nu \ + \frac{1}{2} y_\mu F^\mu \right]$$

  - y_μ Lagrange multiplier
- Gauge fixing $x^\mu = 0$
- Gauge fixed Action

$$S_{fix}(y,v) = \kappa \int_\Sigma d^2\xi \left[(\frac{\eta^{\alpha\beta}}{2} G_{\mu\nu} + \varepsilon^{\alpha\beta} B_{\mu\nu}) v^\mu_\alpha v^\nu_\beta \ + \frac{1}{2} y_\mu F^\mu \right]$$

Buscher T-duality procedure for constant background 3

- Check
  y_μ: $\partial_\alpha v^\mu_\beta - \partial_\beta v^\mu_\alpha = 0 \Longrightarrow v^\mu_\alpha = \partial_\alpha x^\mu \Longrightarrow S_{fix} \to S(x)$
- Elimination of gauge fields on equations of motion produces T-dual Action

$$ {}^\star S(y) = \kappa \int_\Sigma d^2\xi \Big(\frac{\eta^{\alpha\beta}}{2} {}^\star G^{\mu\nu} + \varepsilon^{\alpha\beta\star} B^{\mu\nu}\Big)\partial_\alpha y_\mu \partial_\beta y_\nu \,, $$

Buscher T-duality procedure for constant background 4

- Dual Action ${}^\star S(y)$ has the same form as initial one, but with different background fields

$$ {}^\star S[y] = \kappa \int d^2\xi\ \partial_+ y_\mu\, {}^\star\Pi_+^{\mu\nu}\, \partial_- y_\nu = \frac{\kappa^2}{2} \int d^2\xi\ \partial_+ y_\mu \theta_-^{\mu\nu} \partial_- y_\nu $$

$$ {}^\star G^{\mu\nu} = (G_E^{-1})^{\mu\nu}, \quad {}^\star B^{\mu\nu} = \frac{\kappa}{2}\theta^{\mu\nu} $$

where T-dual background fields

$$ G^E_{\mu\nu} \equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}, \qquad \theta^{\mu\nu} \equiv -\frac{2}{\kappa}(G_E^{-1} B G^{-1})^{\mu\nu} $$

$$ \Pi_\pm \equiv B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}\,, \qquad \theta_\pm^{\mu\nu} \equiv \theta^{\mu\nu} \mp \frac{1}{\kappa}(G_E^{-1})^{\mu\nu} $$

T-duality transformation of variables for constant background

- T-dual transformations

$$v_\pm^\mu \cong \partial_\pm x^\mu \cong -\kappa \Theta_\pm^{\mu\nu} \partial_\pm y_\nu$$

- together with inverse transformation produces
  T-duality transformation of variables

$$\partial_\pm x^\mu \cong -\kappa \theta_\pm^{\mu\nu} \partial_\pm y_\nu \,, \qquad \partial_\pm y_\mu \cong -2\Pi_{\mp\mu\nu} \partial_\pm x^\nu$$

- in canonical form

$$\kappa\, x'^\mu \cong {}^\star\pi^\mu \,, \quad \pi_\mu \cong \kappa y'_\mu \quad -\kappa\, \dot{x}^\mu \cong {}^\star\gamma^\mu_{(0)}(y) \,, \quad \gamma^{(0)}_\mu(x) \cong -\kappa\, \dot{y}_\mu$$

Generalized Buscher approach to T-duality (for weakly curved background) 1

- Background fields

$$G_{\mu\nu} = const\,, \quad B_{\mu\nu} = b_{\mu\nu} + \frac{1}{3} B_{\mu\nu\rho} x^\rho\,, \quad B_{\mu\nu\rho} - infinitesimal$$

- Generalized Buscher procedure:
  - still, there is global symmetry $\delta x^\mu = \lambda^\mu$

$$\delta S = \frac{\kappa}{3} B_{\mu\nu\rho} \lambda^\rho \varepsilon^{\alpha\beta} \int_\Sigma d^2\xi \partial_\alpha (x^\mu \partial_\beta x^\nu) \to 0$$

  - Local symmetry
    - $\partial_\alpha x^\mu \to D_\alpha x^\mu = \partial_\alpha x^\mu + v_\alpha^\mu\,,$
    - $x^\mu \to ? \qquad x^\mu_{inv} = ?$

Generalized Buscher approach to T-duality 2

- Note that $\int_P d\xi^\alpha \partial_\alpha x^\mu = x^\mu(\xi) - x^\mu(\xi_0) \equiv \Delta x^\mu$, path independent
- $\Delta x^\mu_{inv} = \int_P d\xi^\alpha D_\alpha x^\mu = \Delta x^\mu + \Delta V^\mu$
- Line integral of gauge fields

$$\Delta V^\mu = \int_P d\xi^\alpha v^\mu_\alpha$$

- The same procedure as in the case of constant background, but background fields depend on V^μ

Generalized Buscher approach to T-duality 3

- Non-trivial (infinitesimal) variation over $v_\pm^\mu$

$$\delta_V S_{fix} = -\kappa \int d^2\xi \beta^\alpha_\mu(V) \delta v^\mu_\alpha$$

$$\beta^\alpha_\mu(V) = -\frac{1}{3} B_{\mu\nu\rho} \epsilon^{\alpha\beta} V^\rho \partial_\beta V^\nu$$

- Improved equation for $v_\pm^\mu$

$$v_\pm^\mu \cong -\kappa \Theta_\pm^{\mu\nu} [\partial_\pm y_\nu \pm 2\beta_\nu^\mp(V)]$$

- Using this equation

$$V^\mu = -\kappa\theta^{\mu\nu} y_\nu + G_E^{-1\mu\nu} \tilde{y}_\nu \qquad (\dot{y}_\mu = \tilde{y}'_\mu, \dot{\tilde{y}}_\mu = y'_\mu)$$

Source of non-locality

Non-geometry

- Two kinds
  - Locally geometric, globally non-geometric
  - Locally non-geometric
- Features
  - T-dual along non-isometry direction
  - Locally non-geometric
  - non-commutativity
  - non-associativity

Open string Non-commutativity

- Canonical method
  $\gamma^\mu(x,\pi)|_{\partial\Sigma} = 0$ consider as constraints
- Dirac consistency procedure $\Gamma^\mu(\sigma) = 0$
- Solution

$$x^\mu = q^\mu - 2\theta^{\mu\nu} \int d\sigma p_\nu$$

  q^μ and p_μ effective coordinates $\{q^\mu(\sigma), p_\nu(\bar\sigma)\} = \delta^\mu_\nu \delta_S(\sigma,\bar\sigma)$
- Non-commutativity of initial coordinates x^μ
  $\{x^\mu(\sigma), x^\nu(\bar\sigma)\} = 2\theta^{\mu\nu}\theta(\sigma+\bar\sigma)$

Closed string Non-commutativity 1

- Analogy with open string – Canonical method
  But there are no end points, so no boundary conditions
  T-duality solve the problem
- Generalized Buscher procedure in canonical form

$$S = \int d^2\xi [\pi_\mu \dot{x}^\mu - \mathcal{H}(x'^\mu, \pi_\mu, G, B(x))]$$

$$S_{fix} = \int d^2\xi [\pi_\mu v_0^\mu - \mathcal{H}(v_1^\mu, \pi_\mu, G, B(V)) - \kappa(v_0^\mu y'_\mu - v_1^\mu \dot{y}_\mu)]$$

- Only isometry directions, $G, B = const \Longrightarrow \mathcal{H}(v_1^\mu, \pi_\mu, G, B)$, does not depend on V^μ
  $v_0^\mu : \quad \pi_\mu = \kappa y'_\mu$ $\qquad$ y'_μ-does not depend on x^μ
  $\Longrightarrow \quad \{y'_\mu, y'_\nu\} = 0$

Closed string Non-commutativity 2

- Weakly curved background

$$G_{\mu\nu} = const\,, \quad B_{\mu\nu}(V) = \frac{1}{3} B_{\mu\nu\rho} V^\rho, \quad V^\mu = \int (d\xi^0 v_0^\mu + d\xi^1 v_1^\mu)$$

v_0^μ : $\quad \kappa y'_\mu = \pi_\mu - \frac{\kappa}{3} B_{\mu\nu\rho} x'^\nu x^\rho$

y'_μ does depend on x^μ

- T-dual variable y_μ depend on both x^μ and π_μ, source of non-commutativity

$$\{y'_\mu, y'_\nu\} = \frac{1}{\kappa} B_{\mu\nu\rho} x'^\rho \delta$$

$$\{y_\mu(\sigma), y_\nu(\bar\sigma)\} = -\frac{1}{\kappa} B_{\mu\nu\rho} [x^\rho(\sigma) - x^\rho(\bar\sigma)] \theta(\sigma - \bar\sigma)$$

$$\{y_\mu(\sigma + 2\pi), y_\nu(\sigma)\} = -\frac{2\pi}{\kappa} B_{\mu\nu\rho} N^\rho$$

N^ρ – winding number

Closed string Non-associativity

- y'_μ is square function of x^μ
  $\{y_\mu(\sigma), y_\nu(\bar{\sigma})\}$ depend linearly on x^μ
- Non-associativity

  $$\{\{y_\mu(\sigma_1), y_\nu(\sigma_2)\}, y_\rho(\sigma_3)\} - \{y_\mu(\sigma_1), \{y_\nu(\sigma_2), y_\rho(\sigma_3)\} \neq 0$$

- breaking of Jacobi identity

  $$\{\{y_\mu(\sigma_1), y_\nu(\sigma_2)\}, y_\rho(\sigma_3)\} + cyclic(\mu, \sigma_1)(\nu, \sigma_2)(\rho, \sigma_3) \neq 0$$

- The particular form of $V^\mu = -\kappa\,\theta^{\mu\nu} y_\nu + G_E^{-1\mu\nu}\,\tilde{y}_\nu$ implies features of non-geometric theories
- It produces non-commutativity and non-associativity of closed string coordinates

Open string T-duality and non-geometry

- Each term must have its own T-dual

$$\begin{array}{ccccc} S(x) & G_{\mu\nu} & B_{\mu\nu} & A_a^N & A_i^D \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ {}^\star S(y) & {}^\star G^{\mu\nu} & {}^\star B^{\mu\nu} & {}^\star A_D^a & {}^\star A_N^i \end{array}$$

- Coupling for Neumann fields

$$S_{A^N} = 2\kappa \int d\tau (A_a^N \dot{x}^a /_{\sigma=\pi} - A_a^N \dot{x}^a /_{\sigma=0})$$

- Coupling for Dirichlet fields

$$S_{A^D} = 2\kappa \int d\tau (A_i^D (?)^i /_{\sigma=\pi} - A_i^D (?)^i /_{\sigma=0})$$

Zwiebach approach

- Action of closed string theory is invariant under local gauge transformations

$$\delta_\Lambda G_{\mu\nu} = 0\,, \qquad \delta_\Lambda B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

- The open string theory is not invariant

$$\delta_\Lambda S[x] = 2\kappa \int d\tau (\Lambda_a \dot{x}^a /_{\sigma=\pi} - \Lambda_a \dot{x}^a /_{\sigma=0})$$

- To obtain gauge invariant action we should add the term

$$S_{A^N}[x] = 2\kappa \int d\tau (A^N_a \dot{x}^a /_{\sigma=\pi} - A^N_a \dot{x}^a /_{\sigma=0})$$

where newly introduced vector field A^N_a transforms with the same gauge parameter Λ_a

$$\delta_\Lambda A^N_a = -\Lambda_a$$

What is T-dual to local gauge transformations?

- If variation of energy-momentum tensor $T_\pm$ can be written as

$$\delta T_\pm = \{\Gamma, T_\pm\}$$

  then corresponding transformation of background fields is target-space symmetry of the theory.

- $\Gamma \to \Gamma_\Lambda = 2\int d\sigma \Lambda_\mu \kappa x'^\mu$ local gauge transformations

- T-dual to $\kappa x'^\mu$ is π_μ so, T-dual to Γ_Λ is

$$\Gamma_\xi = 2\int d\sigma\, \xi^\mu \pi_\mu$$

  and corresponding transformations have the form of general coordinate transformations

$$\delta_\xi G_{\mu\nu} = -2\,(D_\mu \xi_\nu + D_\nu \xi_\mu)$$

$$\delta_\xi B_{\mu\nu} = -2\,\xi^\rho B_{\rho\mu\nu} + 2\partial_\mu(B_{\nu\rho}\xi^\rho) - 2\partial_\nu(B_{\mu\rho}\xi^\rho)$$

Full gauge invariant action for open string

- Gauge invariant action for open string

$$S_{open}[x] = \kappa \int_{\Sigma} d^2\xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu$$
$$+ \quad 2\kappa \int_{\partial\Sigma} d\tau \Big[A^N_a[x]\dot{x}^a - \frac{1}{\kappa} A^D_i[x] G^{-1ij} \gamma^{(0)}_j(x) \Big]$$

where

$$\delta_\xi A^D_i = -\xi_i$$

- In literature
  - $A^N_a[x]$ is known as massless vector field on Dp-brane
  - $A^D_i[x]$ is known as massless scalar oscillations orthogonal to the Dp-brane

T-dual background fields of the open string

- T-dual background fields in terms of initial ones

$$^\star G^{\mu\nu} = (G_E^{-1})^{\mu\nu}, \quad ^\star B^{\mu\nu} = \frac{\kappa}{2}\theta^{\mu\nu}$$

$$^\star A_D^a(V) = G_E^{-1ab} A_b^N(V)\,, \quad ^\star A_N^i(V) = G^{-1ij} A_j^D(V)$$

- T-duality interchange Neumann with Dirichlet gauge fields

$$V^\mu = -\kappa\,\theta^{\mu\nu} y_\nu + G_E^{-1\mu\nu}\,\tilde{y}_\nu$$

$$\tilde{y}_\mu \equiv -\varepsilon_\alpha{}^\beta \int d\xi^\alpha \partial_\beta y_\mu = \int (d\tau y'_\mu + d\sigma \dot{y}_\mu)$$

$$\dot{\tilde{y}}_\mu = y'_\mu\,, \qquad \tilde{y}'_\mu = \dot{y}_\mu$$

The field strength for non-geometric theories 1

- In geometric theories the field strength for Abelian vector field is simple $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- Because in non-geometric theories the vector field depends on V^μ, we expect that T-dual field strength will contain derivatives with respect to both variables y_μ and $\tilde{y}_\mu$

The field strength for non-geometric theories 2

- We can define field strengths as

$$^\star S_A[y] = {}^\star S_A^D[y] + {}^\star S_A^N[y] = 2\kappa\eta^{\alpha\beta}\int_{\partial\Sigma} d\tau\, {}^\star\mathcal{A}^\mu_\alpha[V]\,\partial_\beta y_\mu$$
$$= \kappa\int_\Sigma d^2\xi\,\partial_+ y_\mu\, {}^\star\mathcal{F}^{\mu\nu}\partial_- y_\nu$$

- Write out expressions for T-dual field strengths ${}^\star\mathcal{F}^{\mu\nu}$ in terms of derivative of T-dual gauge fields ${}^\star\mathcal{A}^a_0(V)$ and ${}^\star\mathcal{A}^a_1(V)$ with respect to variables y_μ and $\tilde{y}_\mu$

$${}^\star\mathcal{F}^{\mu\nu}_{(a)} = \partial^\mu_y\, {}^\star\mathcal{A}^\nu_0(V) - \partial^\nu_y\, {}^\star\mathcal{A}^\mu_0(V) + \partial^\mu_{\tilde{y}}\, {}^\star\mathcal{A}^\nu_1(V) - \partial^\nu_{\tilde{y}}\, {}^\star\mathcal{A}^\mu_1(V)\,,$$
$${}^\star\mathcal{F}^{\mu\nu}_{(s)} = 2\left[\partial^\mu_{\tilde{y}}\, {}^\star\mathcal{A}^\nu_0(V) + \partial^\nu_{\tilde{y}}\, {}^\star\mathcal{A}^\mu_0(V) + \partial^\mu_y {}^\star\mathcal{A}^\nu_1(V) + \partial^\nu_y\, {}^\star\mathcal{A}^\mu_1(V)\right]$$

The field strength for non-geometric theories 3

- The red expression we can consider as a general definition of the field strength
- Beside antisymmetric part ${}^\star\mathcal{F}^{\mu\nu}_{(a)}$ it also contains the symmetric one ${}^\star\mathcal{F}^{\mu\nu}_{(s)}$
- In definition of both parts, derivatives with respect to both T-dual coordinate y_μ and to its double $\tilde{y}_\mu$ contribute
- The unusual form of ${}^\star\mathcal{F}^{\mu\nu}$ is a consequence of two facts:
  1. the T-dual vector field ${}^\star A^a_D(V)$ are not multiplied by $\dot{y}_a$ but with T-dual σ-momentum ${}^\star G^{-1}_{ab}{}^\star\gamma^b_{(0)}$
  2. the T-dual vector fields depend on V^μ which is function on both y_μ and $\tilde{y}_\mu$

Unification of all T-dual theories 1

- Hull had partial success: He united all theories with d T-dualization ($d = 1, 2, \cdots D$)

  This approach has attracted a lot of attention

  But there are D-different formulations

- We still need only one theory, which contains all T-dual theories

Unification of all T-dual theories 2

- Start with T-dual transformation lows along all coordinates

$$\partial_\pm x^\mu = -\kappa \Theta_\pm^{\mu\nu} \partial_\pm y_\nu$$
$$\partial_\pm y_\mu = -2\Pi_{\mp\mu\nu} \partial_\pm x^\nu$$

- Separate parts with ${\varepsilon_\pm}^\pm = \pm 1$ and ${\eta_\pm}^\pm = 1$

$$\pm\partial_\pm y = G_E \partial_\pm x - 2(BG^{-1})\partial_\pm y$$
$$\pm\partial_\pm x = 2(G^{-1}B)\partial_\pm x + G^{-1}\partial_\pm y$$

Unification of all T-dual theories 3

- Rewrite it in doubled space

$$\partial_\pm Z^M \cong \pm\,\Omega^{MN}\mathcal{H}_{NK}\,\partial_\pm Z^K$$

with new coordinates Z^M

$$Z^M = \begin{pmatrix} x^\mu \\ y_\mu \end{pmatrix}$$

where $\mathcal{H}_{MN}$ is generalized metric

$$\mathcal{H}_{MN} = \begin{pmatrix} G^E_{\mu\nu} & -2B_{\mu\rho}(G^{-1})^{\rho\nu} \\ 2(G^{-1})^{\mu\rho}B_{\rho\nu} & (G^{-1})^{\mu\nu} \end{pmatrix}$$

and

$$\Omega^{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Unification of all T-dual theories 4

- T-duality along all coordinates $\Longleftrightarrow$ replacement x^μ with y_μ

$$^\star Z^M = \begin{pmatrix} y_\mu \\ x^\mu \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x^\mu \\ y_\mu \end{pmatrix} = \mathcal{T} Z$$

- Require that T-dual transformation for double coordinates $^\star Z^M$ has the same form as initial one

$$\partial_\pm {}^\star Z^M \cong \pm \Omega^{MN} {}^\star \mathcal{H}_{NK} \, \partial_\pm {}^\star Z^K$$

we find $$^\star \mathcal{H} = \mathcal{T} \mathcal{H} \mathcal{T}$$

Unification of all T-dual theories 5

- T-dualization along arbitrary number of coordinates

$\Longleftrightarrow$ replacement x^a with $y_a \quad a = 1, 2, \cdots d$

$$Z_a^M = \mathcal{T}^{a\,M}{}_N Z^N \qquad \begin{pmatrix} y_a \\ x^i \\ x^a \\ y_i \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1_a & 0 \\ 0 & 1_i & 0 & 0 \\ 1_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1_i \end{pmatrix} \begin{pmatrix} x^a \\ x^i \\ y_a \\ y_i \end{pmatrix}$$

Produce T-dual background fields in complete agreement with Buscher approach

Unification of all T-dual theories 6

- This interpretation of T-duality (as permutation of the coordinates in double space) works for
  - Bosonic string with flat background
  - Bosonic string with weakly curved background
  - Type IIA and Type IIB superstrings
  - Open bosonic string with gauge fields A_a^N and A_i^D
  - Fermionic T-duality

Example: Three torus

- Nontrivial components of the background

$$G_{\mu\nu} = \delta_{\mu\nu}\,, \quad B_{12} = -\frac{1}{2} H x^3$$

- T-duality transformations between these theories

$$\begin{array}{ccccccc}
 & & {}_1S(y_1, x^2, x^3) & \xrightarrow{T^2} & {}_{12}S(y_1, y_2, x^3) & & \\
 & \nearrow^{T^1} & & \nearrow^{T^1} \; \searrow^{T^3} & & \searrow^{T^3} & \\
S(x^1, x^2, x^3) & \xrightarrow{T^2} & {}_2S(x^1, y_2, x^3) & & {}_{13}S(y_1, x^2, y_3) & \xrightarrow{T^2} & {}_{123}S(y_1, y_2, y_3, V^3) \\
 & \searrow^{T^3} & & \nearrow^{T^1} \; \searrow^{T^3} & & \nearrow^{T^1} & \\
 & & {}_3S(x^1, x^2, y_3) & \xrightarrow{T^2} & {}_{23}S(x^1, y_2, y_3) & &
\end{array}$$

- In the literature the theories $S, {}_1S, {}_{12}S$ and ${}_{123}S$ are known as theories with H, f, Q and R fluxes respectively.

  All theories in red are non-geometric (with R fluxes)

Conclusion

- We constructed the theory in double space which contains all T-dual theories (2^D)
  Good candidate for M-theory

  Up to now it only works for bosonic and Type II theories

- Among these 2^D theories there is one geometric (initial one) and $2^D - 1$ non-geometric (All T-dual)
  In general case, maybe some V^μ can turn to function only of y_μ, so that some non-geometric theories can turn to geometric ones

- We should include S-duality