Type I from type IIB superstring theory via boundary conditions

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Outline of the talk

1. Superstrings
2. Model and boundary condition
3. Solution of boundary conditions
4. Concluding remarks

Basic facts

- Strings are object with one spatial dimension. During motion string sweeps a two dimensional surface called world sheet, parametrized by timelike parameter τ and spacelike one $\sigma \in [0\,,\pi]$. There are open and closed strings.
- Demanding presence of fermions in theory, we obtain superstring theory.
- There are five consistent superstring theories.
- Three approaches to superstring theory: NSR (Neveu-Schwarz-Ramond) (world sheet supersymmetry), GS (Green-Schwarz) (space-time supersymmetry) and pure spinor formalism (N. Berkovits, hep-th/0001035).

Superstring theories

1. Type I
   - Unoriented open and closed strings, $N = 1$ supersymmetry, gauge symmetry group $SO(32)$.
2. Type IIA
   - Closed oriented and open strings, $N = 2$ supersymmetry, nonchiral.
3. Type IIB
   - Closed oriented and open strings, $N = 2$ supersymmetry, chiral.
4. Two heterotic theories
   - Closed oriented strings, $N = 1$ supersymmetry, symmetry group either $SO(32)$ or $E_8 \times E_8$.

Type IIB superstring theory

- We use the action of type IIB superstring theory in pure spinor formulation up to the quadratic terms (without ghost terms)

$$S = \kappa \int_\Sigma d^2\xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu$$
$$+ \int_\Sigma d^2\xi \left[-\pi_\alpha \partial_-(\theta^\alpha + \Psi^\alpha_\mu x^\mu) + \partial_+(\bar\theta^\alpha + \bar\Psi^\alpha_\mu x^\mu)\bar\pi_\alpha + \frac{1}{2\kappa}\pi_\alpha F^{\alpha\beta}\bar\pi_\beta \right]$$

- Definitions: $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}$, Ψ^α_μ and $\bar\Psi^\alpha_\mu$ are NS-R fields and $F^{\alpha\beta}$ is R-R field strength. Momenta π_α and $\bar\pi_\alpha$ are canonically conjugated to θ^α and $\bar\theta^\alpha$. All spinors are Majorana-Weyl ones.
- All background fields are constant.

Choice of model - justification

- The choice of action can be justified using general pure spinor type II action within the T-duality formalism (arXiv: 0405072).
- We assume that background fields are independent of x^μ. In mentioned reference, expressions for background fields as well as action are obtained in an iterative procedure as an expansion in powers of θ^α and $\bar{\theta}^\alpha$. Every step in iterative procedure depends on previuous one, so, for mathematical simplicity, we consider only basic (θ and $\bar{\theta}$ independent) components.

Hamiltonian

Hamiltonian is of the following form

$$H_c = \int d\sigma \mathcal{H}_c \,, \quad \mathcal{H}_c = T_- - T_+ \,, \quad T_\pm = t_\pm - \tau_\pm \,,$$

where

$$\begin{aligned} t_\pm &= \mp\frac{1}{4\kappa} G^{\mu\nu} I_{\pm\mu} I_{\pm\nu} \,, \quad I_{\pm\mu} = \pi_\mu + 2\kappa\Pi_{\pm\mu\nu} x'^\nu + \pi_\alpha \psi^\alpha_\mu - \bar{\psi}^\alpha_\mu \bar{\pi}_\alpha \\ \tau_+ &= (\theta'^\alpha + \psi^\alpha_\mu x'^\mu)\pi_\alpha - \frac{1}{4\kappa}\pi_\alpha F^{\alpha\beta}\bar{\pi}_\beta \\ \tau_- &= (\bar{\theta}'^\alpha + \bar{\psi}^\alpha_\mu x'^\mu)\bar{\pi}_\alpha + \frac{1}{4\kappa}\pi_\alpha F^{\alpha\beta}\bar{\pi}_\beta . \end{aligned}$$

Canonical derivation of boundary conditions

- As time translation generator Hamiltonian H_c must have well defined functional derivatives with respect to coordinates $(x^\mu, \theta^\alpha, \bar{\theta}^\alpha)$ and their canonically conjugated momenta $(\pi_\mu, \pi_\alpha, \bar{\pi}_\alpha)$

$$\delta H_c = \delta H_c^{(R)} - \left[\gamma_\mu^{(0)} \delta x^\mu + \pi_\alpha \delta\theta^\alpha + \delta\bar{\theta}^\alpha \bar{\pi}_\alpha\right] \Big|_0^\pi .$$

- The first term is so called regular term. It does not contain τ and σ derivatives of coordinates and momenta variations.
- The second term has to be zero and we obtain boundary conditions. There are various choices of boundary conditions.

Boundary conditions as canonical constraints

- Let $\Lambda^{(0)}$ be a constraint. Then consistency of constraint demands that it is preserved in time

$$\Lambda^{(n)} \equiv \frac{d\Lambda^{(n-1)}}{d\tau} = \left\{ H_c \, , \Lambda^{(n-1)} \right\} \approx 0 \, . \quad (n = 1, 2, \dots)$$

- In all cases we will consider here, this is an infinite set of constraints. Using Taylor expansion we can rewrite this set of constraints in compact σ-dependent form

$$\Lambda(\sigma) = \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \Lambda^{(n)}(\sigma = 0) \, , \, \tilde{\Lambda}(\sigma) = \sum_{n=0}^{\infty} \frac{(\sigma - \pi)^n}{n!} \Lambda^{(n)}(\sigma = \pi) \, .$$

Choice of boundary conditions

- For coordinates x^μ we choose Neumann boundary conditions
$$\gamma_\mu^{(0)}\big|_0^\pi = 0\,.$$
- In order to preserve $N = 1$ SUSY from initial $N = 2$, for fermionic coordinates we choose
$$(\theta^\alpha - \bar\theta^\alpha)\Big|_0^\pi = 0 \implies (\pi_\alpha - \bar\pi_\alpha)\Big|_0^\pi = 0\,.$$
- σ-dependent form of boundary conditions are the second class constraints. Instead Dirac brackets we will solve them.

Solution of boundary conditions

- Solving boundary conditions, we get

$$x^\mu(\sigma) = q^\mu - 2\Theta^{\mu\nu}\int_0^\sigma d\sigma_1 p_\nu + \frac{\Theta^{\mu\alpha}}{2}\int_0^\sigma d\sigma_1 (p_\alpha + \bar{p}_\alpha)\,,$$

$$\theta^\alpha(\sigma) = \eta^\alpha - \Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\mu - \frac{\Theta^{\alpha\beta}}{4}\int_0^\sigma d\sigma_1 (p_\beta + \bar{p}_\beta)\,,$$

$$\bar{\theta}^\alpha(\sigma) = \bar{\eta}^\alpha - \Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\mu - \frac{\Theta^{\alpha\beta}}{4}\int_0^\sigma d\sigma_1 (p_\beta + \bar{p}_\beta)\,,$$

where

$$\eta^\alpha \equiv \frac{1}{2}(\theta^\alpha + \Omega\bar{\theta}^\alpha)\,, \quad \bar{\eta}^\alpha \equiv \frac{1}{2}(\Omega\theta^\alpha + \bar{\theta}^\alpha)\,,$$

$$p_\alpha \equiv \pi_\alpha + \Omega\bar{\pi}_\alpha\,, \quad \bar{p}_\alpha \equiv \Omega\pi_\alpha + \bar{\pi}_\alpha\,.$$

Ω odd $N=1$ supermultiplet

- Up to the constants, tensors multilying the momenta are Ω odd ($\Omega : \sigma \to \sigma$) combinations of T-dual background fields

$$\Theta^{\mu\nu} = \frac{1}{\kappa} {}^{\star}B^{\mu\nu} \,, \quad \Theta^{\mu\alpha} = \frac{1}{2\kappa}\left({}^{\star}\Psi^{\mu\alpha} + {}^{\star}\bar{\Psi}^{\mu\alpha}\right) \,, \quad \Theta^{\alpha\beta} = \frac{1}{2\kappa} {}^{\star}F_s^{\alpha\beta} .$$

- They are $N=1$ supermultiplet.

Effective theory

- Effective theory is initial theory on the solution of boundary conditions. It is of the form

$$
\begin{aligned}
\mathcal{L}^{eff} &= \frac{\kappa}{2} G^{eff}_{\mu\nu} \eta^{ab} \partial_a q^\mu \partial_b q^\nu + \\
&- \pi_\alpha (\partial_\tau - \partial_\sigma) \left[\eta^\alpha + (\Psi_{eff})^\alpha_\mu q^\mu \right] \\
&+ (\partial_\tau + \partial_\sigma) \left[\bar{\eta}^\alpha + (\Psi_{eff})^\alpha_\mu q^\mu \right] \bar{\pi}_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta}_{eff} \bar{\pi}_\beta \, .
\end{aligned}
$$

Effective background fields

- They are of the form

$$G^{eff}_{\mu\nu} = G_{\mu\nu} - 4B_{\mu\rho}G^{\rho\lambda}B_{\lambda\nu}\,,$$

$$(\Psi_{eff})^{\alpha}_{\mu} = \frac{1}{2}\psi^{\alpha}_{+\mu} + B_{\mu\rho}G^{\rho\nu}\psi^{\alpha}_{-\nu}\,, \quad F^{\alpha\beta}_{eff} = F^{\alpha\beta}_{a} - \psi^{\alpha}_{-\mu}G^{\mu\nu}\psi^{\beta}_{-\nu}\,,$$

where the fields $\psi^{\alpha}_{\pm\mu}$ are defined as $\psi^{\alpha}_{\pm\mu} = \psi^{\alpha}_{\mu} \pm \bar{\psi}^{\alpha}_{\mu}$.

Summary

- We start with type IIB superstring theory which background fields are constnt. Boundary conditions are canonically derived.
- Boundary conditions are treated as canonical constraints. Applying canonical approach we solve boundary conditions and express initial variables in terms of the effective ones.
- Effective theory is expressed in terms of Ω even background fields and variables (unoriented string), background fields make $N = 1$ supermultiplet. Consequently, effective theory describes type I superstring.