Noncommutative field theory from angular twist

GST: NEW IDEAS FOR UNSOLVED PROBLEMS III

Nikola Konjik (Faculty of Physics)

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M. Dimitrijević Ćirić, N.K, A. Samsarov, *Noncommutative scalar quasinormal modes of the Reissner–Nordström black hole* Class.Quant.Grav. 35 (2018) no.17, 175005

M. Dimitrijević Ćirić, N.K, M. Kurkov, F. Lizzi, P. Vitale, *Noncommutative field theory from angular twist* arXiv: 1806.06678

Content

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Approaches to NC geometry : $\star$-product, NC spectral triplet, NC vielbien formalism, matrix models,...

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- $(\hat{\mathcal{A}}, \cdot) \to (\mathcal{A}, \star)$

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$$(f \star g)(x) = exp(i\frac{\theta^{\mu\nu}}{2}\frac{\partial}{\partial y^{\mu}}\frac{\partial}{\partial z^{\nu}})f(y)g(z) \mid_{y,z\rightarrow x}$$

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- One more important NC space-time is κ-Minkowski space-time [Lukierski et al, Dimitrijević and Jonke]

$$[x^0 \overset{\star}{,} x^i] = iax^i$$

and all other commutators are zero

Twist formalism

- A well defined way to deform symmetries (symm. alg. g with generators t^a)
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$$[t^a, t^b] = if^{ab}_{\ \ c}t^c, \quad \Delta(t^a) = t^a \otimes 1 + 1 \otimes t^a,$$

$$\epsilon(t^a) = 0, \quad S(t^a) = -t^a.$$

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- Abelian twist

$$\mathcal{F}^{-1} = e^{i\theta^{AB} X_A \otimes X_B}$$

where X_A and X_B are commuting vector fields and θ^{AB} is an antisymmetric constant matrix

- Deformation of differential calculus

$$f \star g = \mu \mathcal{F}^{-1}(f \otimes g) \qquad \omega_1 \wedge_\star \omega_2 = \wedge \mathcal{F}^{-1}(\omega_1 \otimes \omega_2).$$

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- Action of the vector field on the differential forms is given by the Lie derivative along that vector field $X_A \triangleright \omega = \ell_{X_A} \omega$
- Twisted Hopf algebra is

$$\begin{aligned} [t^a, t^b] &= i f^{ab}_{\ \ c} t^c, \\ \Delta_{\mathcal{F}}(t^a) &= \mathcal{F} \Delta(t^a) \mathcal{F}^{-1}, \\ \epsilon(t^a) &= 0, \quad S_{\mathcal{F}}(t^a) = \mathrm{f}^\alpha S(\mathrm{f}_\alpha) S(t^a) S(\bar{\mathrm{f}}^\beta) \mathrm{f}_\beta. \end{aligned}$$

where is

$$\mathcal{F} = \mathrm{f}^\alpha \otimes \mathrm{f}_\alpha, \quad \mathcal{F}^{-1} = \bar{\mathrm{f}}^\alpha \otimes \bar{\mathrm{f}}_\alpha,$$

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$$\mathcal{F} = e^{-\frac{ia}{2}\left(\partial_z\otimes(x\partial_y - y\partial_x) - (x\partial_y - y\partial_x)\otimes\partial_z\right)} = e^{-\frac{ia}{2}\left(\partial_z\otimes\partial_\varphi - \partial_\varphi\otimes\partial_z\right)},$$

gives commutation relations between coordinates

$$[z \stackrel{\star}{,} x] = -iay, \quad [z \stackrel{\star}{,} y] = iax.$$

or

$$[z \stackrel{\star}{,} \varphi] = ia \;\; \text{or} \;\; [z \stackrel{\star}{,} e^{i\varphi}] = iae^{i\varphi}$$

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- Twist similar to $\kappa - Minkowski$ $[t \stackrel{\star}{,} x^i] = iax^i$ but more simpler
- Vector fields in the angular twist are Poincare generatore, while in the $\kappa - Minkowski$ we have dilatation generator

Angular noncommutativity

- Product of two plane waves is

$$e^{-ip\cdot x} \star e^{-iq\cdot x} = e^{-i(p+_\star q)\cdot x}$$

where is $p +_\star q = R(q_3)p + R(-p_3)q$ and

$$R(t) \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\frac{at}{2}\right) & \sin\left(\frac{at}{2}\right) & 0 \\ 0 & -\sin\left(\frac{at}{2}\right) & \cos\left(\frac{at}{2}\right) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Angular noncommutativity

- $e^{-ip\cdot x} \star e^{-iq\cdot x} \star e^{-ir\cdot x} = e^{-i(p +_\star q +_\star r)\cdot x}$ gives

$$p +_\star q +_\star r = R(r_3 + q_3)p + R(-p_3 + r_3)q + R(-p_3 - q_3)r$$

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- General case

$$p^{(1)} +_\star \dots +_\star p^{(N)} = \sum_{j=1}^{N} R\left(-\sum_{1\leq k<j} p_3^{(k)} + \sum_{j<k\leq N} p_3^{(k)}\right) p^{(j)}$$

- Conservation of momentum is broken!

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- $\Delta^{\mathcal{F}} P_2 =$
  $P_2 \otimes \cos\left(\frac{a}{2}P_3\right) + \cos\left(\frac{a}{2}P_3\right) \otimes P_2 - P_1 \otimes \sin\left(\frac{a}{2}P_3\right) + \sin\left(\frac{a}{2}P_3\right) \otimes P_1$

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- Suppose that a field (or a state) $\phi_p$is an eigenvector of the momentum operator P_μ with the eigenvalue p_μ: $P_\mu \phi_p = p_\mu \phi_p$
- Then $P_\mu(\phi_p \star \phi_q) = \mu_\star\{\Delta^{\mathcal{F}} P_\mu(\phi_p \otimes \phi_q)\} = (p +_\star q)(\phi_p \star \phi_q)$

Scalar field theory

$$S[\phi] = \int_{R^4} d^4x \left(\frac{1}{2}\partial_\mu \phi(x) \star \partial^\mu \phi(x) + \frac{1}{2} m^2 \phi(x) \star \phi(x) + \frac{\lambda}{4!} \phi(x)^{\star 4} \right)$$

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In momentum space, action has the form

$$S[\phi] = \int_{R^4 \times R^4} dp\, dq\, \frac{1}{2} \left(-p_\mu q^\mu \widetilde{\phi}(p) \widetilde{\phi}(q) + m^2 \widetilde{\phi}(p) \widetilde{\phi}(q) \right) \delta^{(4)}\left(p +_\star q\right)$$

$$+ \frac{1}{(2\pi)^4} \frac{\lambda}{4!} \int_{(R^4)^{\times 4}} dp\, dq\, dr\, ds\, \widetilde{\phi}(p) \widetilde{\phi}(q) \widetilde{\phi}(r) \widetilde{\phi}(s) \delta^{(4)}\left(p +_\star q +_\star r +_\star s\right)$$

Scalar field theory

All NC corrections are in the delta function with 4 terms because

$$\delta^{(4)}(p +_\star q) = \delta^{(4)}(R(q_3)p + R(-p_3)q) = \delta^{(4)}(R(q_3)(p+q)) = \delta^{(4)}(p+q)$$

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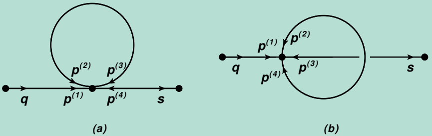
p(2)
p(3)
q
p(1)
p(4)
s
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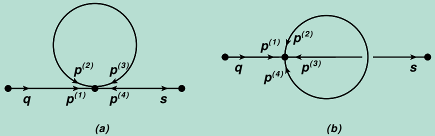
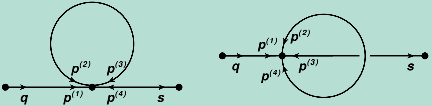


Value of the UV divergent part of the planar diagram

$$\mathrm{Pl(a)} = \frac{1}{q^2 + m^2} \cdot \frac{1}{s^2 + m^2} \cdot \delta^{(4)}(q - s)\pi^2 \left(\Lambda^2 - m^2 \log \frac{\Lambda}{\mu} \right)$$



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Value of the UV divergent part of the non-planar diagram

$$\mathrm{NPl(b)} = \frac{1}{q^2+m^2}\,\frac{1}{s^2+m^2}\,\delta(q_0-s_0)\,\delta(q_3-s_3)\,\frac{\pi^2}{2\left(\sin\left(\frac{aq_3}{2}\right)\right)^2}\ln\left(\frac{\Lambda}{\mu}\right)$$

UV/IR mixing in the Moyal case

- Planar diagrams are without NC corrections

$$\Gamma_{Pl} = \frac{g^2}{96\pi^2}(\Lambda^2 - m^2 ln(\frac{\Lambda^2}{m^2}) + fin.\ part)$$

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$$\Gamma_{NPl} = \frac{g^2}{48\pi^2}(\Lambda^2_{eff} - m^2 ln(\frac{\Lambda^2_{eff}}{m^2}) + fin.\ part)$$

where is $\Lambda^2_{eff} = \frac{1}{\frac{1}{\Lambda^2} - p \circ p}$ and $p \circ p = |p_\mu \theta^2_{\mu\nu} p_\nu|$

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- For $p \circ p = finite$ the NPI diagram is finite
- The NPI diagram is divergent for $p \to 0$ (UV/IR mixing) or for $\theta \to 0$

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- In 3D we have finite non-planar diagrams

Particle decay

- Application of the $\star$-sum of momenta to the kinematics of particles decay

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- For this chapter, we will change type of NC: we will change z-coordinate with time coordinate
- We will look at kinematically decay of one particle in the rest to the two other particles. Momentum law conservation is

$$p +_\star (-q) +_\star (-r) = a(-q_0 - r_0) \cdot p - a(-p_0 - r_0) \cdot q - a(-p_0 + q_0) \cdot r$$

Particle decay

- These four equations are

$$M = E_q + E_r$$

$$0 = q_z + r_z$$

$$0 = \cos\left(\frac{a}{2}(M + E_r)\right)q_x - \sin\left(\frac{a}{2}(M + E_r)\right)q_y + \cos\left(\frac{aE_r}{2}\right)r_x - \sin\left(\frac{aE_r}{2}\right)r_y$$

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- NC effects are obviousy in the xy-plane

- Equations for x and y plane gives

$$0 = R(\frac{aE_r}{2})R(\frac{aM}{2})\vec{q}_{xy} + R(\frac{aE_r}{2})\vec{r}_{xy} = R(\frac{aM}{2})\vec{q}_{xy} + \vec{r}_{xy}$$

$\vec{q}_{xy}$-projection of the momentum on the xy-plane
R-rotational matrix which rotate one of the opposite momenta for angle $\frac{aM}{2}$

Example: $Z^0 \to \mu^+ + \mu^-$

$M(Z) \approx 90 GeV \;\; m(\mu) \approx 105 MeV \;\; |\vec{p}_{\mu^+}| = |\vec{p}_{\mu^-}| \approx 45 GeV$
$\angle(\vec{p}_{\mu^-}, \vec{p}_{\mu^+}) = \pi - \frac{aM}{2}$
$a \sim (10\,TeV)^{-1} \Rightarrow \frac{aM}{2} \sim 5 \cdot 10^{-3}$ and
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- Boosts will change angle between particles: different angles from different reference systems

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- Simple Lie algebra NC
- Connection between deformed Hopf algebra and $\star$-sums
- Momentum and boost conservation laws broken
- UV/IC "mixing"
- Phenomenological aspects of particle decay
- Future work: better understanding of the connection between deformed conservation laws and deformed Hopf algebra, spectrum of QNMs with some numerical methods