Electrodynamics and Yang-Mills theory in $SO(2,3)_\star$ model of Noncommutative Gravity

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GRAVITY and STRING THEORY: NEW IDEAS FOR UNSOLVED PROBLEMS III

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Short overview

1. Canonical deformation of continuous structure of spacetime; algebra of coordinates: $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$; representation of the algebra *via* NC Moyal-Weyl $\star$-product.
2. $SO(2,3)$ gauge theory of gravity: unification of spin-connection and vierbein; gravity emerges only after gauge SB $SO(2,3) \to SO(1,3)$
3. NC deformation of GR:

$$\widehat{S} = -\frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\Big(R + \theta^{\alpha\beta}\theta^{\gamma\delta}\Big(\frac{7}{2l^4}R_{\alpha\beta\gamma\delta} - \frac{15}{16l^4}T_{\alpha\beta}{}^{\rho}T_{\gamma\delta\rho} + \ldots\Big)\Big)$$

4. NC deformation of Minkowski space; interpretation of θ-constant noncommutativity; *Fermi inertial coordinates*
5. Dirac spinors are incorporated; NC corrections are *linear* in $\theta^{\alpha\beta}$ and they modify the propagator and dispersion relation of a free electron; helicity dependent energy levels; birefringence effect

1. D. Gočanin and V. Radovanović, *Dirac field and gravity in NC $SO(2,3)_\star$ model*, Eur. Phys. J. C, **78** (2018) 195.
2. M. Dimitrijević-Ćirić, D. Gočanin, N. Konjik and V. Radovanović, *Noncommutative Electrodynamics from $SO(2,3)_\star$ Model of Noncommutative Gravity*, Eur. Phys. J. C, **78** (2018) 548.
3. M. Dimitrijević-Ćirić, D. Gočanin, N. Konjik and V. Radovanović, *Yang-Mills Theory in the $SO(2,3)_\star$ Model of Noncommutative Gravity* (under revision)

Next step: gauge fields in $SO(2,3)$ framework

Unifying gravitational field with *Maxwell* $U(1)$ gauge field and *Yang-Mills* $SU(N)$ gauge field within $SO(2,3)$ model.

Gauge group $SO(2,3) \times SU(N)$. Generators T_I of $SU(N)$ group are hermitian, traceless and they satisfy the (anti)commutation relations: $[T_I, T_J] = if_{IJK}T_K$ and $\{T_I, T_J\} = d_{IJK}T_K$; normalization $Tr(T_I T_J) = \delta_{IJ}$.

Unifying gauge potential:

$$\Omega_\mu = \frac{1}{2}\omega_\mu^{AB} M_{AB} \otimes \mathbb{I} + \mathbb{I} \otimes A_\mu^I T_I \equiv \frac{1}{4}\omega_\mu^{ab}\sigma_{ab} - \frac{1}{2l}e_\mu^a \gamma_a + A_\mu^I T_I$$

Field strength:

$$\mathbb{F}_{\mu\nu} = \partial_\mu \Omega_\nu - \partial_\nu \Omega_\mu - i[\Omega_\mu, \Omega_\nu] = \frac{1}{2}F_{\mu\nu}^{AB} M_{AB} \otimes \mathbb{I} + \mathbb{I} \otimes \mathcal{F}^I T_I$$

$$F_{\mu\nu}^{ab} = R_{\mu\nu}^{ab} - \frac{2}{l^2} e^a_{(\mu} e^b_{\nu)}, \qquad F_{\mu\nu}^{a5} = \frac{1}{l} T_{\mu\nu}^a \qquad \mathcal{F}_{\mu\nu}^I = 2\partial_{(\mu} A_{\nu)}^I + gf^{IJK}A^J A^K$$

Commutative model

Commutative (un-deformed) action:

$$S_A = -\frac{1}{16l} Tr \int d^4x \; \varepsilon^{\mu\nu\rho\sigma} \Big\{ f \mathbb{F}_{\mu\nu} D_\rho\phi D_\sigma\phi\phi + \frac{i}{3!} f^2 D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi\phi \Big\} + h.c.$$

1. action is hermitian and possesses $SO(2,3) \times SU(N)$ gauge symmetry
2. we don't have metric *a priori* and thus *no Hodge dual operation* $\rightarrow$ auxiliary field $f = \frac{1}{2} f^{AB,I} M_{AB} \otimes T_I$
3. auxiliary field $\phi = \phi^A \Gamma_A$ is used for symmetry breaking: $\phi^a = 0$ and $\phi^5 = l$; covariant derivative after SB: $(D_\mu\phi)^a = e_\mu^a$ and $(D_\mu\phi)^5 = 0$
4. it reduces to the canonical kinetic action for $SU(N)$ gauge field in *curved spacetime* after the SB:

$$S_A = -\frac{1}{4} \int d^4x \, \sqrt{-g} \; g^{\mu\rho} g^{\nu\sigma} \mathcal{F}^I_{\mu\nu} \mathcal{F}^I_{\rho\sigma}$$

NC-deformed action

ordinary commutative product $\to$ noncommutative Moyal-Weyl $\star$-product

$$\widehat{S}_A = -\frac{1}{16l} T_r \int d^4x \; \varepsilon^{\mu\nu\rho\sigma} \Big\{ \widehat{f} \star \widehat{\mathbb{F}}_{\mu\nu} \star D_\rho \widehat{\phi} \star D_\sigma \widehat{\phi} \star \widehat{\phi} + \frac{i}{6} \widehat{f} \star \widehat{f} \star D_\mu \widehat{\phi} \star D_\nu \widehat{\phi} \star D_\rho \widehat{\phi} \star D_\sigma \widehat{\phi} \star \widehat{\phi} \Big\} + h.c.$$

Deformed action is hermitian and invariant under $SO(2,3)_\star \times SU(N)_\star$ gauge transformations. Symmetry is noncommutatively preserved!

This action can now be perturbatively expanded in powers of $\theta^{\alpha\beta}$ and the Seiberg-Witten map insures that, order-by-order, this expansion possesses ordinary $SO(2,3) \times SU(N)$ gauge symmetry of the original undeformed action S_A.

After the symmetry breaking, the leading term in NC perturbative expansion turns out to be linear in $\theta^{\alpha\beta}$:

$$\widehat{S}_A^{(1)} = -\frac{\theta^{\alpha\beta}}{16} \int d^4x \; e \; d_{IJK} \; g^{\mu\rho} g^{\nu\sigma} \left(\mathcal{F}^I_{\alpha\beta} \mathcal{F}^J_{\mu\nu} \mathcal{F}^K_{\rho\sigma} - 4 \mathcal{F}^I_{\alpha\mu} \mathcal{F}^J_{\beta\nu} \mathcal{F}^K_{\rho\sigma} \right)$$

Fermions

NC-deformed $SO(2,3)_\star \times SU(N)_\star$ invariant spinor action before SB:

$$\widehat{S}_\Psi = \frac{i}{12}\int d^4x\, \varepsilon^{\mu\nu\rho\sigma} \Big[\widehat{\bar{\Psi}} \star (D_\mu\widehat{\phi}) \star (D_\nu\widehat{\phi}) \star (D_\rho\widehat{\phi}) \star (D_\sigma\widehat{\Psi}) - (D_\sigma\widehat{\bar{\Psi}}) \star (D_\mu\widehat{\phi}) \star (D_\nu\widehat{\phi}) \star (D_\rho\widehat{\phi}) \star \widehat{\Psi}\Big] + \widehat{S}_m$$

Linear NC correction of Yang-Mills theory in curved spacetime after the SB:

$$\widehat{S}^{(1)}_\Psi = \theta^{\alpha\beta}\int d^4x\, e\, \bar{\Psi}\Big(\mathcal{A}_{\alpha\beta}{}^{\rho\sigma}\mathcal{D}_\rho\mathcal{D}_\sigma + \mathcal{B}_{\alpha\beta}{}^{\sigma}\mathcal{D}_\sigma + \mathcal{C}_\alpha\mathcal{D}_\beta + \mathcal{D}_{\alpha\beta}\Big)\Psi$$

New terms: $\bar{\Psi}\sigma_\alpha{}^\sigma\mathcal{D}_\beta\mathcal{D}_\sigma\Psi$, $\bar{\Psi}R_{\alpha\beta}{}^{\rho\sigma}\gamma_\rho\mathcal{D}_\sigma\Psi$, $\bar{\Psi}T_{\alpha\beta}{}^{\sigma}\mathcal{D}_\sigma\Psi$, $\bar{\Psi}\sigma_{\alpha\beta}\Psi$, ...

$SO(1,3)\times SU(N)$ covariant derivative:

$$(\mathcal{D}_\sigma\Psi)_i = \nabla_\sigma\psi_i - iA^I_\sigma(T_I)_{ij}\psi_j$$

NC Yang-Mills theory in Minkowski space

Residual effects of noncommutativity in flat spacetime produce new types of coupling in Yang-Mills theory:

$$\widehat{S}_{flat} = \int d^4x \left\{ i\bar{\Psi}\mathcal{D}\Psi - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^I_{\rho\sigma} \right.$$

$$+\theta^{\alpha\beta}\left[-\frac{1}{2l}\bar{\Psi}\sigma_\alpha{}^\sigma\mathcal{D}_\beta\mathcal{D}_\sigma\Psi + \frac{7i}{24l^2}\varepsilon_{\alpha\beta}{}^{\rho\sigma}\bar{\Psi}\gamma_\rho\gamma_5\mathcal{D}_\sigma\Psi + \frac{3i}{4}\bar{\Psi}\mathcal{F}_{\alpha\beta}\mathcal{D}\Psi \right.$$

$$-\frac{i}{2}\bar{\Psi}\mathcal{F}_{\alpha\sigma}\gamma^\sigma\mathcal{D}_\beta\Psi - \frac{1}{6l^3}\bar{\Psi}\sigma_{\alpha\beta}\Psi + \frac{1}{4l}\bar{\Psi}\mathcal{F}_{\alpha\beta}\Psi$$

$$\left.\left. -\frac{1}{16}d_{IJK}\, g^{\mu\rho}g^{\nu\sigma}\left(\mathcal{F}^I_{\alpha\beta}\mathcal{F}^J_{\mu\nu}\mathcal{F}^K_{\rho\sigma} - 4\mathcal{F}^I_{\alpha\mu}\mathcal{F}^J_{\beta\nu}\mathcal{F}^K_{\rho\sigma}\right)\right]\right\}$$

Dispersion relation for "free" electrons

NC Dirac equation: $\left(i\partial\!\!\!/ - m + A\!\!\!/ + \theta^{\alpha\beta}\mathcal{M}_{\alpha\beta}\right)\psi = 0$

For "free" electron, $[H, \mathbf{p}] = 0 \Rightarrow$ plane wave ansatz $\psi(x) = u(\mathbf{p})e^{-ip\cdot x}$

For $\mathbf{p} = (0, 0, p_z)$ and $[\hat{x}^1, \hat{x}^2] = i\theta^{12}$, $\theta^{12} \equiv \theta \neq 0$, we have four independent energy functions:

$$
\begin{aligned}
E_{1,2} &= E_{\mathbf{p}} \mp \left[\frac{m^2}{12l^2} - \frac{m}{3l^3}\right]\frac{\theta}{E_{\mathbf{p}}} + \mathcal{O}(\theta^2) \\
E_{3,4} &= -E_{\mathbf{p}} \pm \left[\frac{m^2}{12l^2} - \frac{m}{3l^3}\right]\frac{\theta}{E_{\mathbf{p}}} + \mathcal{O}(\theta^2)
\end{aligned}
$$

with classical energy $E_{\mathbf{p}} = \sqrt{m^2 + p_z^2}$. $\Rightarrow$ **Zeeman effect!**

Important observation: Electron's energy depends on its *helicity*!
Background NC space behaves as a *birefringent* medium for electrons propagating in it.

Electron in magnetic field - NC Landau levels

Special case: electron in magnetic field $\mathbf{B} = B\mathbf{e}_z$

NC-deformed energy levels:

$$\begin{aligned} E_{n,s} &= \sqrt{p_z^2 + m^2 + (2n+s+1)B} \\ &- \theta\left[\frac{mM^3 s}{E_{n,s}^{(0)}} + \frac{M^3 Bs}{E_{n,s}^{(0)}(E_{n,s}^{(0)} + m)}(2n+s+1) - \frac{B^2}{2E_{n,s}^{(0)}}(2n+s+1)\right] \end{aligned}$$

Non-relativistic limit, $B \ll m^2$, for an electron confined in NC plane:

$$E_{n,s} = m_{eff} + \frac{2n+s+1}{2m} B_{eff} - \frac{(2n+s+1)^2}{8m^3} B_{eff}^2$$

$$m_{eff} = m - s\theta M^3, \; B_{eff} = (B + \theta B^2), \; M^3 = \left[\frac{m}{12l^2} - \frac{1}{3l^3}\right].$$

Inverse of the canonical deformation parameter θ plays the role of an effective magnetic field $b = \theta^{-1}$.

NC correction to magnetic dipole moment of an electron:

$$\mu_{n,s} = -\frac{\partial E_{n,s}}{\partial B} = -\mu_B(2n+1+s)(1+\theta B)$$

Further development of $SO(2,3)_\star$ model

1. **Renormalizability**: Minimal NC Electrodynamics,

$$\widehat{S} = \int d^4x \, \widehat{\bar{\psi}} \star (i\gamma^\mu D_\mu - m)\widehat{\psi} - \frac{1}{4} \int d^4x \, \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}$$

   is not renormalizabile. Minimal NC Yang-Mills is one-loop renormalizabile to first order in θ.
2. **NC Standard Model**: introducing (complex) scalars
3. **NC SUGRA**: Introduce local SUSY; spin-3/2 gravitino field; *orthosymplectic* $OSp(1,4)$ group;

$$[P^\mu, Q_\alpha] \propto \frac{1}{l}\sigma^\mu_{\alpha\dot{\alpha}}\bar{Q}^{\dot{\alpha}}$$

$$\{Q_\alpha, Q_\beta\} \propto \frac{1}{l}(\sigma^{\mu\nu})_{\alpha\beta}M_{\mu\nu} \quad \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} \propto \sigma^\mu_{\alpha\dot{\beta}}P_\mu$$

4. **NC quantum Hall effect**: How noncommutativity influences the phenomenology of the Hall effect?
5. **Quark-gluon plasma in NC spacetime**: How noncommutativity influences the phenomenology of quark-gluon plasma in the early stages of the Universe?

The End