LAGRANGIAN OF A SCALAR FIELD - $\mathcal{L}(X,\phi)$

- In general case – any function of a scalar field ϕ and kinetic energy $X \equiv \frac{1}{2}\partial_\mu\phi\partial_\nu\phi$.
  - Canonical field, potential $V(\phi)$

$$\mathcal{L}(X,\phi) = BX - V(\phi),$$

  - Non-canonical models

$$\mathcal{L}(X,\phi) = BX^n - V(\phi),$$

  - Dirac-Born-Infeld (DBI) Lagrangian

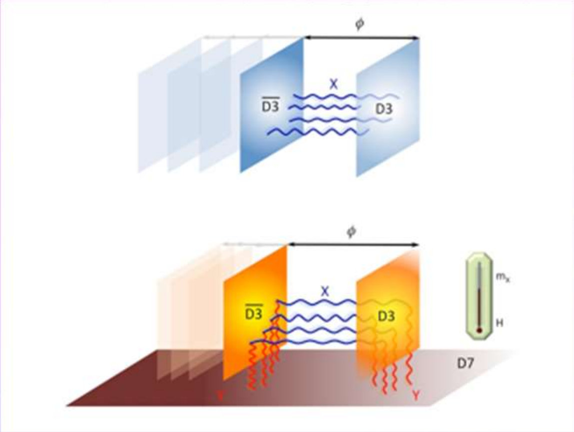
$$\mathcal{L}(X,\phi) = -\frac{1}{f(\phi)}\sqrt{1-2f(\phi)X} - V(\phi),$$

    - Special case – tachyonic $\mathcal{L}(X,\phi) = -V(\phi)\sqrt{1-2\lambda X},$

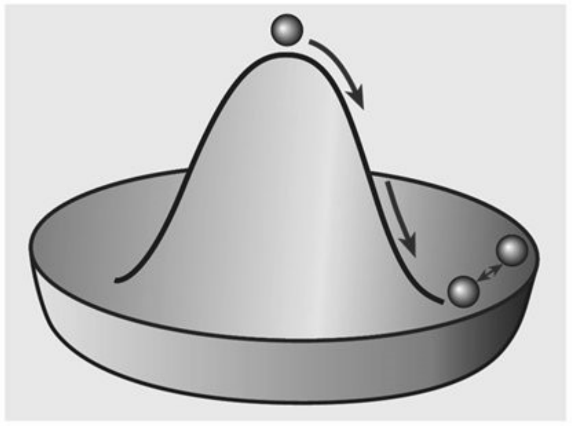
TACHYONS

- Traditionally, the word tachyon was used to describe a hypothetical particle which propagates faster than light (Sommerfeld 1904 ?).
- In modern physics this meaning has been changed
  - The effective tachyonic field theory was **proposed** by A. Sen
  - **String theory**: states of quantum fields with imaginary mass (i.e. negative mass squared)
  - It **was believed**: such fields permitted propagation faster than light
  - However it **was realized** that the imaginary mass creates an instability and tachyons spontaneously decay through the process known as tachyon condensation

TACHYION FIELDS



- No classical interpretation of the ”imaginary mass”
  - The instability: The potential of the tachyonic field is initially at a local maximum rather than a local minimum (like a ball at the top of a hill)
  - A small perturbation - forces the field to roll down towards the local minimum.
  - Quanta are not tachyon any more, but rather an ”ordinary” particle with a positive mass.



REFERENCES: TACHYONS` QUANTIZATION – (NON)ARCHIMEDEAN SPACES

- D.D. Dimitrijevic, G.S. Djordjevic and Lj. Nesic,

  Real and p-Adic aspects of quantization of tachyons, in Mathematical, theoretical and phenomenological challenges beyond the standard model, World scientific (2005) 197-207, (eds) G. S. Djordjevic, Lj. Nesic and J. Wess

- D.D. Dimitrijevic, G.S. Djordjevic and Lj. Nesic

  Fortschritte der Physik, **56** No. 4-5 (2008) 412-417

- Dragoljub D. Dimitrijevic, G. S. Dj and Milan Milosevic

  Classicalization and Quantization of Tachyon-like Matter on (non)Archimedean Spaces, Rom.Rep.Phys. 68 (2016) No 1, 5

TACHYON INFLATION

- Consider the tachyonic field T minimally coupled to Einstein's gravity with action

$$S = -\frac{1}{16\pi G}\int \sqrt{-g}\, R d^4x + \int \sqrt{-g}\, \mathcal{L}(T, \partial_\mu T) d^4x$$

- Where *R* is Ricci scalar, and Lagrangian and Hamiltionian for tachyon potential $V(T)$ are

$$\mathcal{L} = -V(T)\sqrt{1 - g^{\mu\nu}\partial_\mu T \partial_\nu T},$$

$$\mathcal{H} = \frac{V(T)}{\sqrt{1 - g^{\mu\nu}\partial_\mu T \partial_\nu T}}.$$

- Homogenous and isotropic space, FRW metrics

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)d\vec{x}^2, \qquad c = 1$$

TACHYON INFLATION

- As well as for a standard scalar field $P = \mathcal{L}$ i $\rho = \mathcal{H}$, however:

$$\mathcal{L} = -V(T)\sqrt{1 - \dot{T}^2},$$

$$\mathcal{H} = \frac{V(T)}{\sqrt{1-\dot{T}^2}}.$$

- Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\frac{V}{(1-\dot{T}^2)^{1/2}}.$$

Reduced Planck mass

$$M_P = \sqrt{\frac{1}{8\pi G}}$$

- Energy momentum conservation equation, $\dot{\rho} = -3H(P + \rho)$, takes a form

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V'}{V} = 0.$$

TACHYON INFLATION

$$\tau = t/T_0$$

$$x = \frac{T}{T_0}, \quad U(x) = \frac{V(x)}{\sigma}, \quad \widetilde{H} = \frac{H}{T_0}.$$

- Non-dimensional equations

Energy-momentum conservation eq.

$$\ddot{x} + \kappa\sqrt{3U(x)(1-\dot{x}^2)^{3/2}}\dot{x} + \frac{(1-\dot{x}^2)}{U(x)}\frac{dU(x)}{dx} = 0$$

$$\widetilde{H}^2 = \frac{\kappa^2}{3}\frac{U(x)}{\sqrt{1-\dot{x}^2}}$$

Friedmann eq.

$$\dot{\widetilde{H}} = -\frac{\kappa^2}{2}(\tilde{P} + \tilde{\rho})$$

Friedmann acceleration eq.

- Dimensionless constant $\kappa^2 = \frac{\sigma T_0^2}{M_{Pl}^2}$, a choice of a constant σ (brane tension) was motivated by string theory

$$\sigma = \frac{M_s^4}{g_s(2\pi)^3}.$$

CONDITION FOR TACHYON INFLATION

- General condition for inflation

$$\frac{\ddot{a}}{a} \equiv \widetilde{H}^2 + \dot{\widetilde{H}} = \frac{\kappa^2}{3} \frac{U(x)}{\sqrt{1-\dot{x}^2}} \left(1 - \frac{3}{2}\dot{x}^2\right) > 0.$$

- Slow-roll conditions

$$\ddot{x} \ll 3\widetilde{H}\dot{x}, \quad \dot{x}^2 \ll 1.$$

- Equations for slow-roll inflation

$$\widetilde{H}^2 \simeq \frac{\kappa^2}{3} U(x),$$

$$\dot{x} \simeq -\frac{1}{3\widetilde{H}} \frac{U'(x)}{U(x)}.$$

INITIAL CONDITION FOR TACHYON INFLATION

- Basic ideas, problems …. (Steer, Vernizzi 2004)
- Slow-roll parameters

$$\epsilon_1 \simeq \frac{1}{2\kappa^2}\frac{{U'}^2}{U^3}, \quad \epsilon_2 \simeq \frac{1}{\kappa^2}\left(-2\frac{U''}{U^2} + 3\frac{{U'}^2}{U^3}\right).$$

- Number of e-folds

$$N(x) = \kappa \int_{x_i}^{x_e} \frac{U(x)^2}{|U'(x)|} dx$$

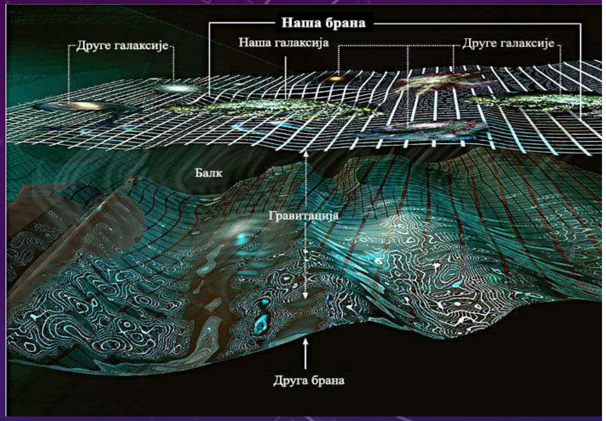
$x_i = x(\tau_i)$

$x_e = x(\tau_e)$

BRANEWORLD UNIVERSE

- Braneworld universe is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk.
- N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429** (1998)
- L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 (RS I)
- L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690 (RS II)
- 1998 ADD / 2000 DGP

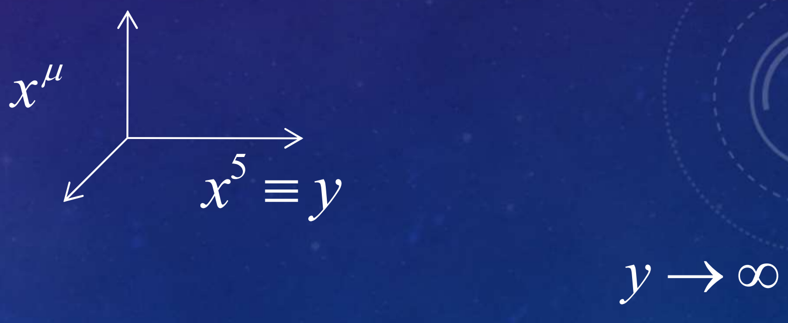
D-BRANES, COSMOLOGY WITH EXTRA DIMENISONS



- 1999 – RSI and RSII
- We will consider the Randall-Sundrum scenario with a braneworld embedded in a 5-dim asymptotically Anti de Sitter space (AdS5)
- One of the simplest models
- Two branes with opposite tensions are placed at some distance in 5 dimensional space
- RSI model – observer reside on the brane with negative tension, distance to the 2nd brane corresponds to the Newtonian gravitational constant
- RSII – observer is placed on the positive tension brane, 2nd brane is pushed to infinity

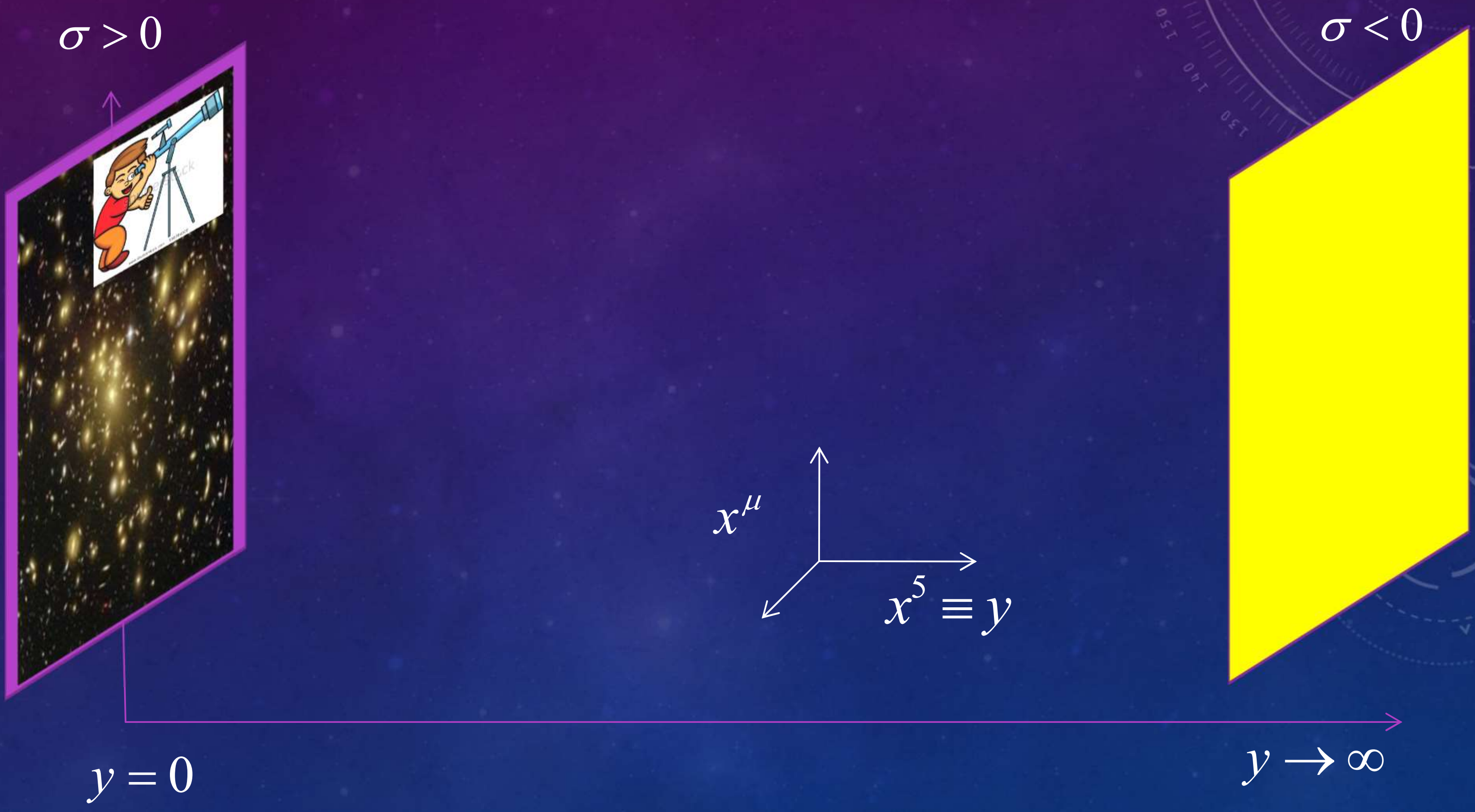
RSI MODEL

- Observers reside on the negative tension brane at $y = l$.
- The coordinate position $y = l$ of the negative tension brane
- serves as a compactification radius so that the effective
- compactification scale is $\mu_c = 1/l$.



RSII MODEL

- Observers reside on the positive tension brane at
- $y = 0$ and the negative tension brane is pushed off to infinity in the fifth dimension.



RSII MODEL

- The space is described by Anti de Siter metric

$$ds^2_{(5)} = e^{-2ky} g^{\mu\nu} dx^\mu dx^\nu - dy^2.$$

- Extended RSII model includes radion backreaction

$$ds^2_{(5)} = G_{ab} dX^a dX^b = \frac{1}{k^2 z^2}\left[\left(1 + k^2 z^2 \eta(x)\right) g^{\mu\nu} dx^\mu dx^\nu - \frac{1}{\left(1 + k^2 z^2 \eta(x)\right)^2} dz^2\right],$$

$k = \frac{1}{\ell}$ ← AdS curvature radius

Radion field

- Total action

$$S = S_{bulk} + S_{br} + S_{mat}.$$

- After integrating out 5th dimension…

RSII MODEL

- Action for a 3-brane moving in bulk

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}\right) + S_{\rm br},$$

Canonicali normalized radion field

$$\eta = \sinh^2\left(\sqrt{\frac{4\pi G}{3}}\Phi\right)$$

- Action for the brane

Brane tension

$$S_{\rm br} = -\sigma\int d^4x\sqrt{-\det g^{\rm ind}_{\mu\nu}}$$

$$= -\int d^4x\sqrt{-g}\,\frac{\sigma}{k^4\Theta^4}(1+k^2\Theta^2\eta)^2\sqrt{1-\frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{(1+k^2\Theta^2\eta)^3}}$$

Tachyon field $\Theta = e^{ky}\,/\,k$

- Without radion $\Phi = 0$

$$S^{(0)}_{\rm br} = -\int d^4x\sqrt{-g}\,\frac{\lambda}{\Theta^4}\sqrt{1-g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}, \qquad \lambda = \frac{\sigma}{k^4}$$

- Total Lagrangian

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} - \frac{\lambda\psi^2}{\Theta^4}\sqrt{1-\frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{\psi^3}}, \qquad \psi = 1+k^2\Theta^2\eta.$$

N. Bilic, G.B. Tupper, CEJP 12 (2014) 147–159.

RSII MODEL

- In flat space, FRW metrics

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2).$$

- Hamiltonian equations

$$\Pi_\Phi^\mu \equiv \frac{\partial L}{\partial \Phi_{,\mu}}, \quad \Pi_\Theta^\mu \equiv \frac{\partial L}{\partial \Theta_{,\mu}}.$$

- The Hamiltonian

$$\mathcal{H} = \frac{1}{2}\Pi_\Phi^2 + \frac{\lambda\psi^2}{\Theta^4}\sqrt{1 + \Pi_\Theta^2\,\Theta^8/(\,\lambda^2\psi)}$$

RSII MODEL

- The Hamiltonian equations

$$\begin{aligned}\dot{\Phi} &= \frac{\partial\mathcal{H}}{\partial\Pi_{\Phi}}\\ \dot{\Theta} &= \frac{\partial\mathcal{H}}{\partial\Pi_{\Theta}}\\ \dot{\Pi}_{\Phi} + 3H\Pi_{\Phi} &= -\frac{\partial\mathcal{H}}{\partial\Phi}\\ \dot{\Pi}_{\Theta} + 3H\Pi_{\Theta} &= -\frac{\partial\mathcal{H}}{\partial\Theta}\end{aligned}$$

- The modified Friedman equation

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\mathcal{H}\left(1+\frac{2\pi G}{3k^2}\mathcal{H}\right)}.$$

- Combining with a continuity equation $\dot{\mathcal{H}} + 3H(\mathcal{H} + \mathcal{L}) = 0$ it leads to the second Friedman equation

$$\dot{H} = -4\pi G(\mathcal{H} + \mathcal{L})\left(1+\frac{4\pi G}{3k^2}\mathcal{H}\right)$$

NONDIMENSIONAL EQUATIONS

- Substitutions:
$$h = H / k,$$
$$\phi = \Phi / (k\sqrt{\lambda}),\ \pi_\phi = \Pi_\Phi / (k^2\sqrt{\lambda})),$$
$$\theta = k\Theta,\quad \pi_\theta = \Pi_\Theta / (k^4\lambda)$$

$$\dot{\phi} = \pi_\phi$$

$$\dot{\theta} = \frac{\theta^4 \psi \pi_\theta}{\sqrt{1 + \theta^8 \pi_\theta^2 / \psi}}$$

$$\dot{\pi}_\phi = -3h\pi_\phi - \frac{\psi}{2\theta^2} \frac{4 + 3\theta^8 \pi_\theta^2 / \psi}{\sqrt{1 + \theta^8 \pi_\theta^2 / \psi}} \eta'$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{\psi}{\theta^5} \frac{4 - 3\theta^{10} \eta \pi_\theta^2 / \psi}{\sqrt{1 + \theta^8 \pi_\theta^2 / \psi}}$$

$$\left.\begin{aligned} \dot{h} &= -\frac{\kappa^2}{2}(\bar{\rho} + \bar{p})\left(1 + \frac{\kappa^2}{6}\bar{\rho}\right) \\ \dot{N} &= h \end{aligned}\right\}$$

Additional equations, solved in parallel

Nondimensional constant →
$$\kappa^2 = 8\pi\lambda G k^2$$

Hubble parameter →
$$h \equiv \frac{\dot{a}}{a} = \sqrt{\frac{\kappa^2}{3}\bar{\rho}\left(1 + \frac{\kappa^2}{12}\bar{\rho}\right)}$$

$$\psi = 1 + \theta^2 \eta,$$

$$\eta = \sinh^2\left(\sqrt{\frac{\kappa^2}{6}}\phi\right),$$

$$\eta' = \frac{d\eta}{d\phi} = \sqrt{\frac{\kappa^2}{6}} \sinh\left(\sqrt{\frac{2\kappa^2}{3}}\phi\right),$$

Preassure →
$$\bar{p} = \frac{1}{2}\dot{\phi}^2 - \frac{\psi^2}{\theta^4}\sqrt{1 - \dot{\theta}^2 / \psi^3},$$

Energy density →
$$\bar{\rho} = \frac{1}{2}\dot{\phi}^2 + \frac{\psi^2}{\theta^4}\frac{1}{\sqrt{1 - \dot{\theta}^2 / \psi^3}}$$

INITIAL CONDITIONS FOR RSII MODEL

- Initial conditions – from a model without radion field
- “Pure” tachyon potential $V(\Theta) = \frac{\lambda}{\Theta^4}$
- Hamiltonian $\mathcal{H} = \frac{\lambda}{\Theta^4}\sqrt{1 + \Pi_\Theta^2\,\Theta^8/\lambda^2}\,.$
- Nondimensional equation

$$\begin{aligned}\dot{\theta} &= \frac{\theta^4\pi_\theta}{\sqrt{1+\theta^8\pi_\theta^2}}\\ \dot{\pi}_\theta &= -3h\pi_\theta + \frac{4}{\theta^5\sqrt{1+\theta^8\pi_\theta^2}}\,.\end{aligned}$$

ESTIMATION OF INITIAL CONDITIONS

- The end of inflation $\varepsilon_1 \approx 1$, tj. $\kappa^2/\theta_f^4 \ll 1$ → RSII modification can be neglected

$$\epsilon_1(\theta_\mathrm{f}) \simeq \epsilon_2(\theta_\mathrm{f}) \simeq \frac{8\theta_\mathrm{f}^2}{\kappa^2} \simeq 1, \qquad h(\theta_\mathrm{f}) \simeq \frac{8}{\sqrt{3\kappa}}.$$

- Number of e-folds

$$N \simeq \frac{\kappa^2}{8\theta_0^2}\left(1 + \frac{\kappa^2}{36\theta_0^4}\right).$$

- Number of e-folds (standard tachyon inflation)

$$N_\mathrm{st.tach} \simeq \frac{\kappa^2}{8\theta_0^2} - 1.$$

- Huge difference in number of e-folds → RSII extends the period of inflation!!!

$$\kappa^2 = 5, \theta_0 = 0,25 \quad \Rightarrow \begin{cases} N_\mathrm{st.tach} \simeq 9 \\ N \simeq 330 \end{cases}$$

OBSERVATIONAL PARAMETERS

- Scalar spectral index n_s and tensor-to-scalar ratio r (the first order of parameters ε_{i})

$$r = 16\varepsilon_1(t_i),$$

$$n_s = 1 - 2\varepsilon_1(t_i) - \varepsilon_2(t_i)$$

- The second order of parameters ε_{i} → different

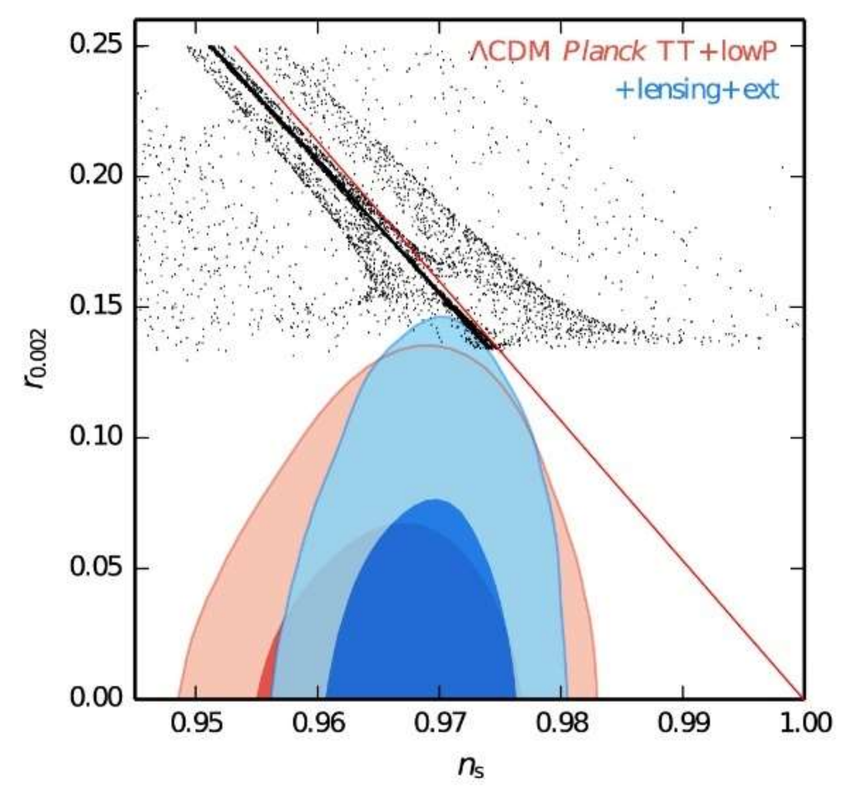
$$r = 16\varepsilon_1\left(1 + C\varepsilon_2 - 2\alpha\varepsilon_1\right),$$
$$n_s = 1 - 2\varepsilon_1 - \varepsilon_2 - \left[2\varepsilon_1^2 + \left(2C + 3 - 2\alpha\right)\varepsilon_1\varepsilon_2 + C\varepsilon_2\varepsilon_3\right].$$

- Always constant $C \simeq -0{,}72$, however constant $\alpha = \frac{1}{6}$ for tachyon inflation in standard cosmology, and $\alpha = \frac{1}{12}$ for Randall-Sundrum cosmology

NUMERICAL RESULTS

OBSERVATIONAL PARAMETERS $(\boldsymbol{n_s},\boldsymbol{r})$, $U(x)=\frac{1}{x^4}$

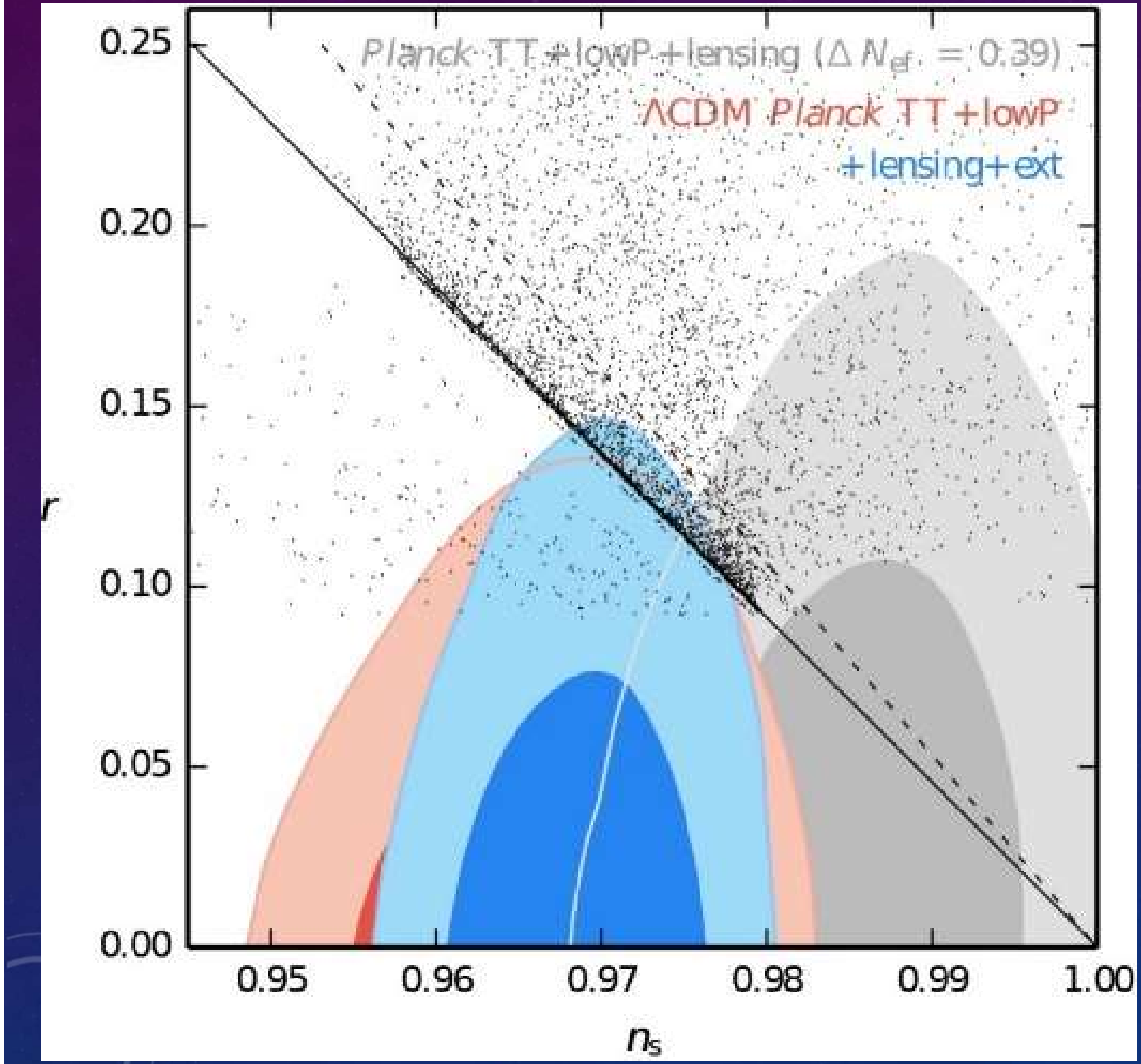
- Diagram with observational constraints from Planck 2015.
- The dots represent the calculation in the tachyon model for various N, $\boldsymbol{\kappa}$
- 35% of calculated results for pairs of free parameters is represented in the plot.
- **Red solid line** represents the slow-roll approximation of the standard tachyon model with inverse quartic potential. $r=\frac{16}{3}(1-n_s)$.



$$45 \leq N \leq 120$$

$$1 \leq \kappa \leq 25$$

OBSERVATIONAL PARAMETERS (n_s, r), RSII MODEL



- Free parameters from the interval:

$$60 \leq N \leq 120$$

$$1 \leq \kappa \leq 12$$

$$0 \leq \phi_0 \leq 0{,}5$$

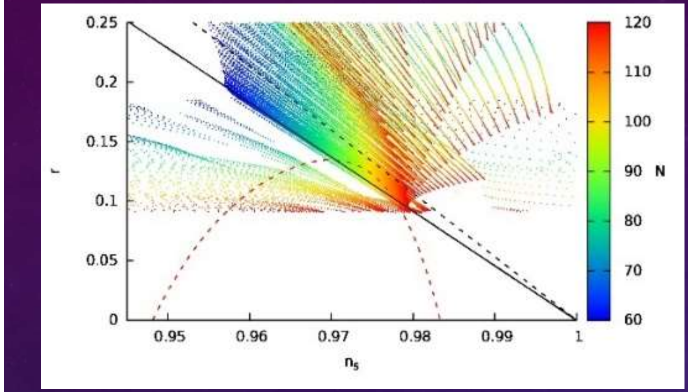
- Approximate relation:
  - RS model

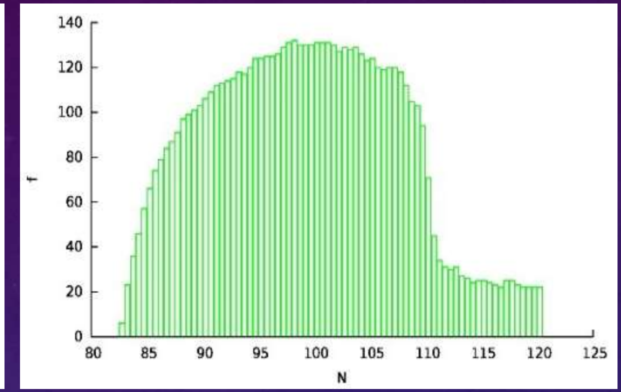
$$r = \frac{32}{7}(1 - n_s)$$ ———

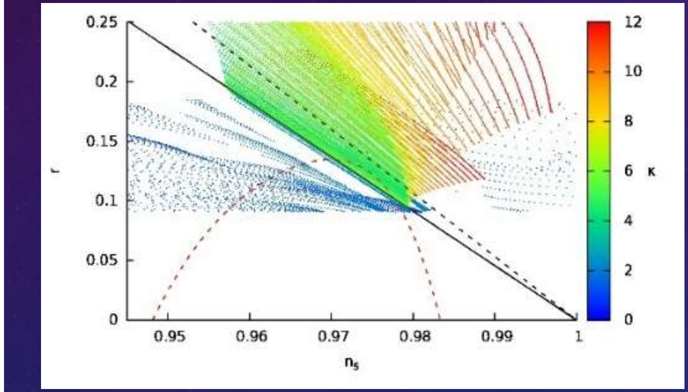
  - Tachyon model (FRW)

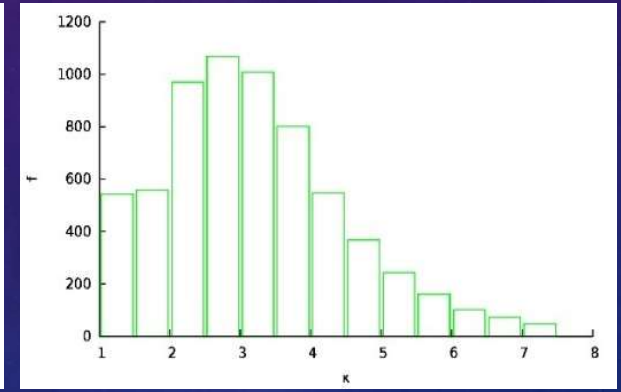
$$r = \frac{16}{3}(1 - n_s)$$ - - -

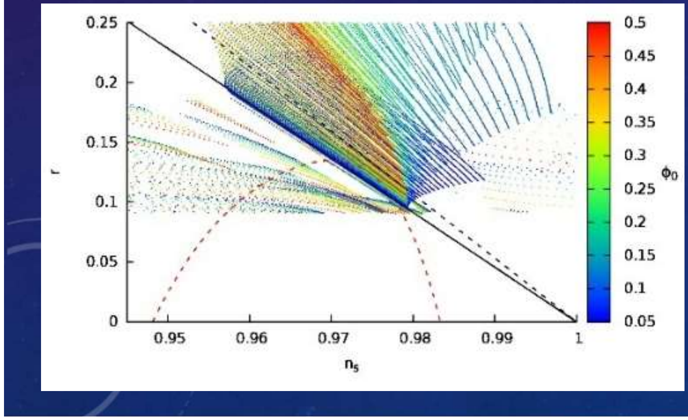
(n_s, r) AS A FUNCTION OF N, κ, ϕ_0

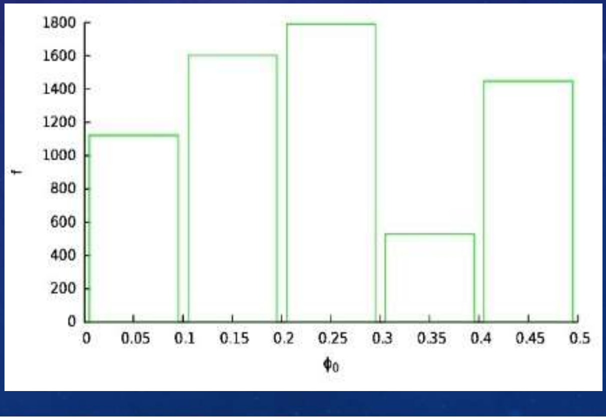












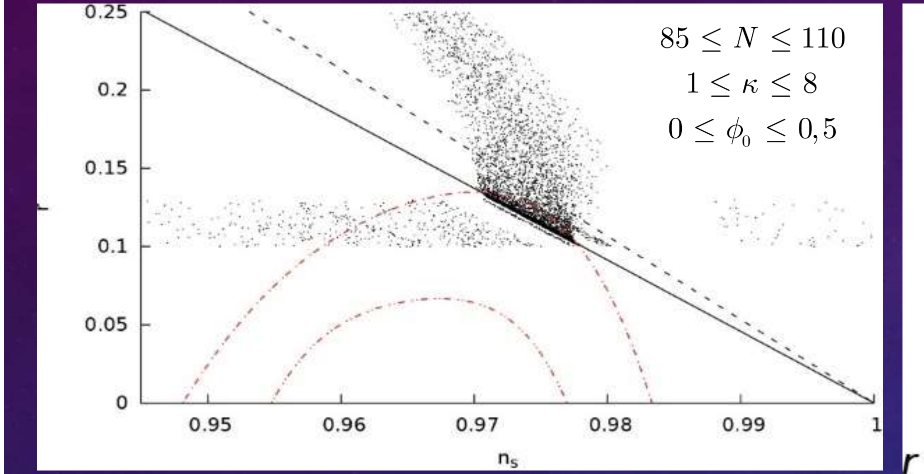
$$60 \leq N \leq 120, \quad \Delta N = 0,5$$

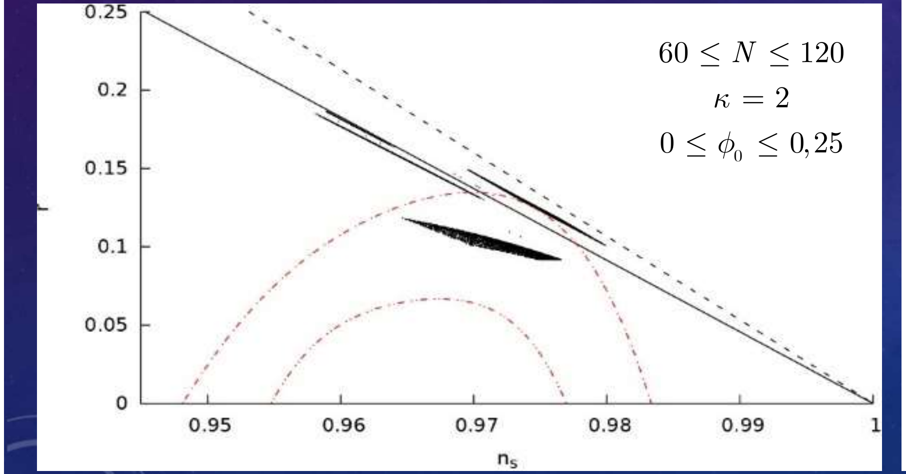
$$1 \leq \kappa \leq 12, \quad \Delta\kappa = 0,5$$

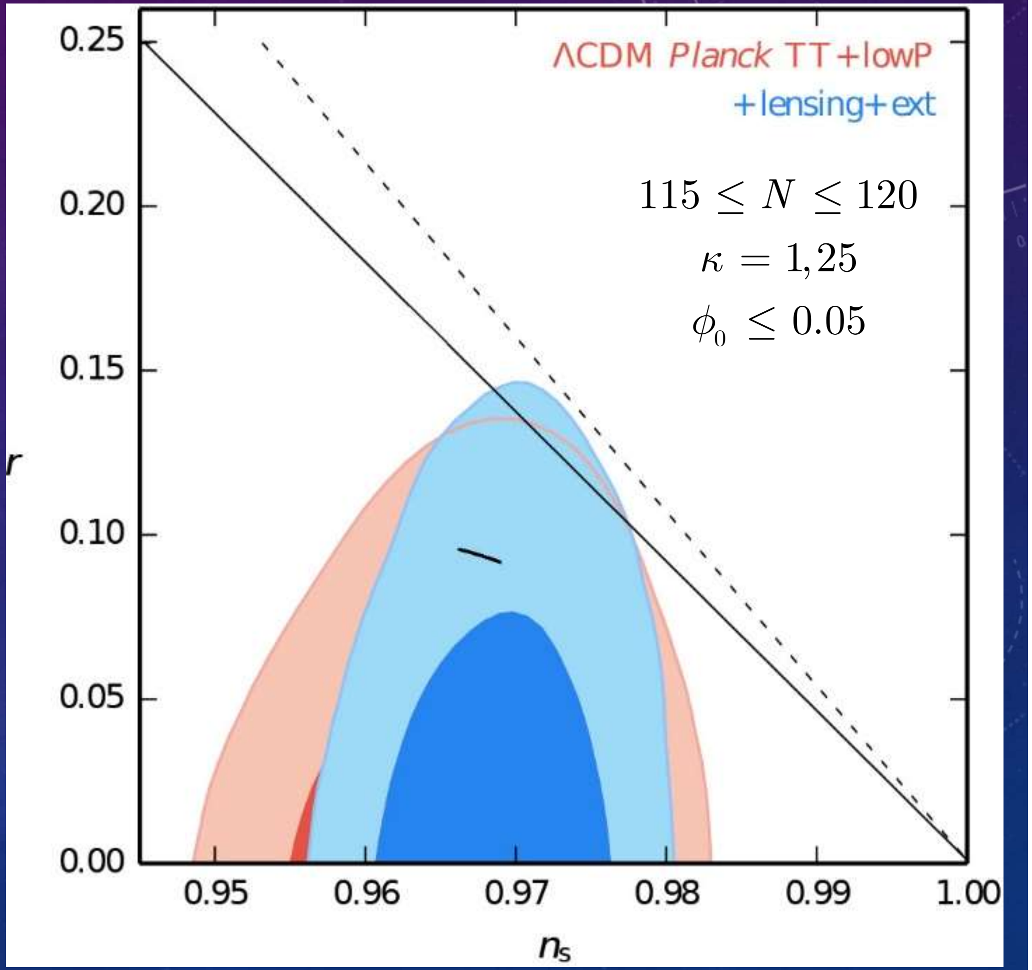
$$0 \leq \phi_0 \leq 0,5, \quad \Delta\phi_0 = 0.05$$

- 65% is plotted, 12% in 2σ range

THE BEST FITTING RESULTS (n_s, r)







TACHYON WITH AN INVERSE POWER-LAW POTENTIAL IN A BRANEWORLD COSMOLOGY

Here, we study a quite similar tachyon cosmological model based on the dynamics of a 3-brane in the bulk of the second Randall-Sundrum model extended to more general warp functions, i.e. with a selfinteracting scalar

- As a consequence, on the observer brane G is modified to be the scale dependent four-dimensional gravitational constant. A power law warp factor generates an inverse power-law potential V ~ 1/ϕ

TACHYON WITH AN INVERSE POWER-LAW POTENTIAL IN A BRANEWORLD COSMOLOGY

- Introducing a combined dimensionless coupling

$$\kappa^2 = \frac{8\pi G_5}{k}\sigma = \frac{8\pi G_{\mathrm{N}}}{k^2}\sigma$$

- and dimensionless functions, in the same way as it was done for the previous models, we obtain the following set of equations

$$\dot{\varphi} = \frac{\chi^4 \pi_\varphi}{\sqrt{1+\chi^8 \pi_\varphi^2}} = \frac{\pi_\varphi}{\rho}$$

$$\dot{\pi}_\varphi = -3h\pi_\varphi + \frac{4\chi_{,\varphi}}{\chi^5\sqrt{1+\chi^8\pi_\varphi^2}}$$

- Where

$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(\chi_{,\varphi} + \frac{\kappa^2}{12}\rho\right)}, \qquad \text{and} \qquad \chi_{,\varphi} = \frac{\partial \chi}{\partial \varphi}$$

- We analyze in detail the tachyon with potential

$$\chi(\varphi) = \varphi^{n/4}$$

TACHYON WITH AN INVERSE POWER-LAW POTENTIAL IN A BRANEWORLD COSMOLOGY

- Following the similar procedure as in the previous RSII model, for a given N and κ inital condition for the tachyon field can be obtained from the slow-roll condition

$$N \simeq \frac{2n}{(4n-1)\epsilon_1(\varphi_\mathrm{i})} - \frac{3n+1}{2(3n-1)}$$

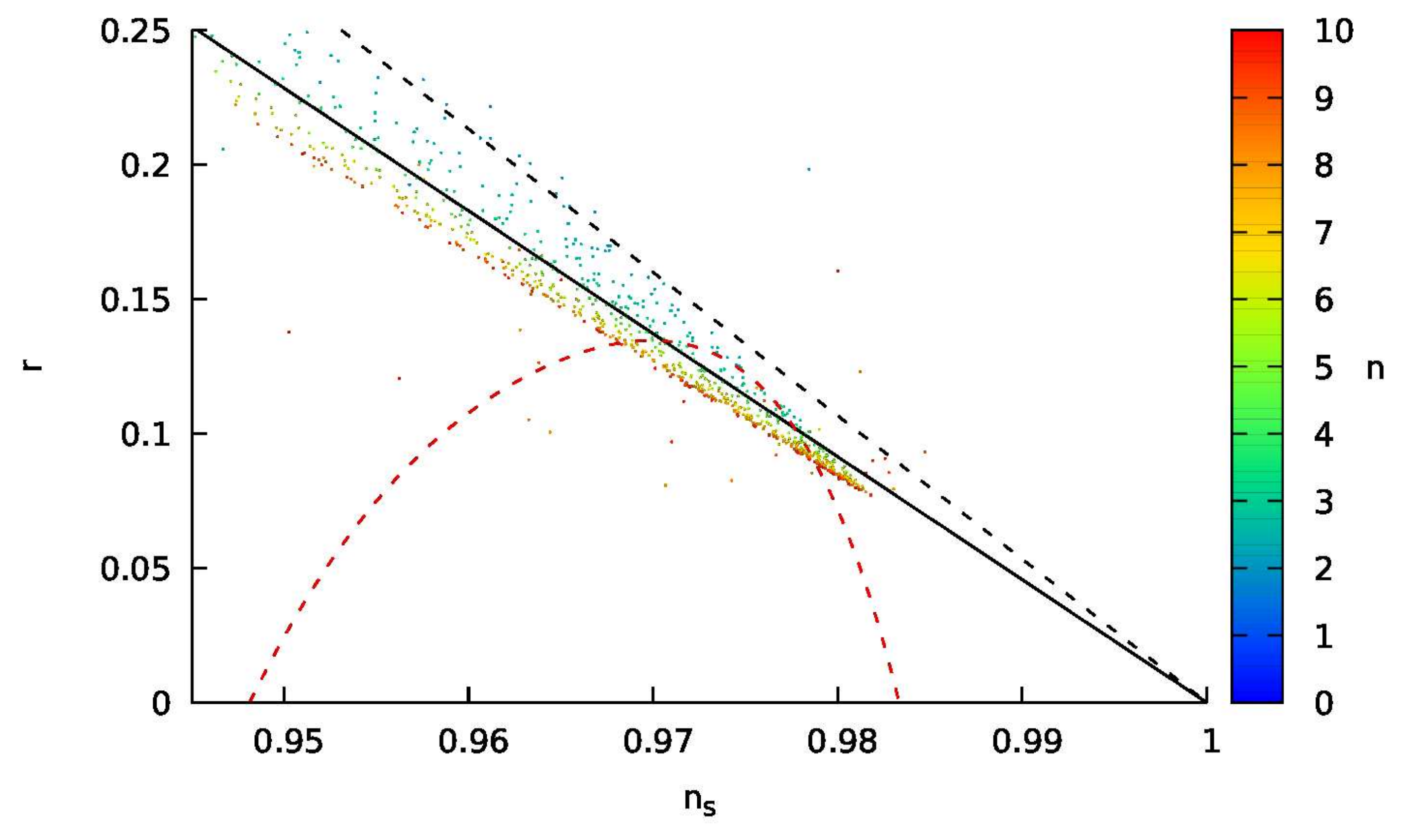
- Where

$$\epsilon_1(\varphi_\mathrm{i}) \simeq 192 \frac{\chi^6(\theta_\mathrm{i})\chi^2_{,\theta}(\theta_\mathrm{i})}{\kappa^4}$$

- Here, we find the critical value

``dust vs quasi de Sitter``. $n > \frac{1}{3}$

NEW RESULTS



- 1000 randomly chosen values of free parameters (N, κ, n)

$45 \leq N \leq 120$

$0.5 \leq \kappa \leq 10$

$0.5 \leq n \leq 10$

ONGOING RESEARCH - RSII AND HOLOGRAPHIC COSMOLOGY

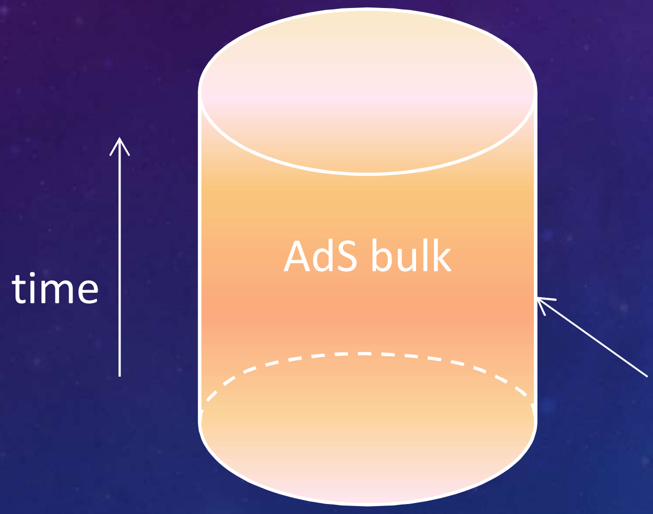
- Here we present unpolished results and ongoing work

Connection with AdS/CFT

AdS/CFT correspondence is a holographic duality between gravity in $d+1$-dim space-time and quantum CFT on the d-dim boundary. Original formulation stems from string theory:

Equivalence of 3+1-dim
$\mathcal{N}=4$ Supersymmetric YM Theory
and string theory in $AdS_5 \times S_5$

J. Maldacena, Adv. Theor. Math. Phys. **2** (1998)



Conformal
Boundary
at $z=0$

Examples of CFT:
quantum electrodynamics,
$\mathcal{N}=4$ Super YM gauge theory,
massless scalar field theory,
massless spin ½ field theory

Holographic cosmology

We start from AdS-Schwarzschild static coordinates and make the coordinate transformation $t = t(\tau, z),\ r = r(\tau, z)$ The line element will take a general form

$$ds^2_{(5)} = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2) = \frac{\ell^2}{z^2}\left[n^2(\tau,z)d\tau^2 - a^2(\tau,z)d\Omega_k^2 - dz^2\right]$$

Imposing the boundary conditions at *z*=0:

$$n(\tau,0) = 1, \quad a(\tau,0) = a_{\mathrm{h}}(\tau)$$

we obtain the induced metric at the boundary in the FRW form

$$ds^2_{(0)} = g^{(0)}_{\mu\nu}dx_\mu dx_\nu = d\tau^2 - a_{\mathrm{h}}^2(\tau)d\Omega_k^2$$

Solving Einstein's equations in the bulk one finds

$$a^2 = a_{\mathrm{h}}^2\left[\left(1-\frac{\mathcal{H}_{\mathrm{h}}^2 z^2}{4}\right)^2 + \frac{1}{4}\frac{\mu z^4}{a_{\mathrm{h}}^4}\right], \qquad n = \frac{\dot{a}}{\dot{a}_{\mathrm{h}}},$$

where $\mathcal{H}_{\mathrm{h}}^2 = H_{\mathrm{h}}^2 + \frac{\kappa}{a_{\mathrm{h}}^2}$ $\quad H_{\mathrm{h}} = \frac{\dot{a}_{\mathrm{h}}}{a_{\mathrm{h}}}$ Hubble rate at the boundary

P.S. Apostolopoulos, G. Siopsis and N. Tetradis, Phys. Rev. Lett. **102**, (2009)

Comparing the exact solution with the expansion

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + z^4 g_{\mu\nu}^{(4)} + \cdots$$

we can extract $g_{\mu\nu}^{(2)}$ and $g_{\mu\nu}^{(4)}$. Then, using the de Haro et al expression for T^{CFT} we obtain

$$\left\langle T^{\mathrm{CFT}}_{\mu\nu} \right\rangle = t_{\mu\nu} + \frac{1}{4}\left\langle T^{\mathrm{CFT}\,\alpha}{}_{\alpha} \right\rangle g^{(0)}_{\mu\nu}$$

The second term is the conformal anomaly

$$\left\langle T^{\mathrm{CFT}\,\alpha}{}_{\alpha} \right\rangle = \frac{3\ell^3}{16\pi G_5}\frac{\ddot{a}_{\mathrm{h}}}{a_{\mathrm{h}}}\mathcal{H}_{\mathrm{h}}^2$$

The first term is a traceless tensor with non-zero components

$$t_{00} = -3t^i_i = \frac{3\ell^3}{64\pi G_5}\left(\mathcal{H}_{\mathrm{h}}^4 + \frac{4\mu}{a_{\mathrm{h}}^4} - \frac{\ddot{a}_{\mathrm{h}}}{a_{\mathrm{h}}}\mathcal{H}_{\mathrm{h}}^2\right)$$

Hence, apart from the conformal anomaly, the CFT dual to the time dependent asymptotically AdS$_5$ bulk metric is a conformal fluid with the equation of state $p_{\mathrm{CFT}} = \rho_{\mathrm{CFT}}/3$ where ,

$$\rho_{\mathrm{CFT}} = t_{00} \qquad p_{\mathrm{CFT}} = -t^i_i$$

from Einstein’s equations on the boundary we obtain the holographic Friedmann equation

$$\mathcal{H}_{\mathrm{h}}^{2}=\frac{8\pi G_{\mathrm{N}}}{3}\rho_{\mathrm{h}}+\boxed{\frac{\ell^{2}}{4}\left(\mathcal{H}_{\mathrm{h}}^{4}+\frac{4\mu\ell}{a_{\mathrm{h}}^{4}}\right)}$$

quadratic deviation

dark radiation

Kiritsis, JCAP **0510** (2005) ; Apostolopoulos et al, Phys. Rev. Lett. **102**, (2009)

The second Friedmann equation can be derived from energy-momentum conservation

$$\frac{\ddot{a}_{\mathrm{h}}}{a_{\mathrm{h}}}\left(1-\frac{\ell^{2}}{2}\mathcal{H}_{\mathrm{h}}^{4}\right)+\mathcal{H}_{\mathrm{h}}^{2}=\frac{4\pi G_{\mathrm{N}}}{3}\left(\rho_{\mathrm{h}}-3p_{\mathrm{h}}\right)$$

quadratic deviation

where $\rho_{\mathrm{h}}=T_{00}^{\mathrm{matt}}\,,\ \ p_{\mathrm{h}}=-T^{\mathrm{matt}\,i}{}_{i}$

Holographic map

The time dependent bulk spacetime with metric

$$ds^2_{(5)} == \frac{\ell^2}{z^2}\left[n^2(\tau,z)d\tau^2 - a^2(\tau,z)d\Omega_k^2 - dz^2\right]$$

may be regarded as a *z*-foliation of the bulk with FRW cosmology on each *z*-slice. In particular:

at $z=z_{\rm br}$: RSII cosmology, at $z=0$: holographic cosmology.

A map between *z*-cosmology and *z*=0-cosmology can be constructed using

$$a^2 = a_{\rm h}^2\left[\left(1-\frac{\mathcal{H}_{\rm h}^2 z^2}{4}\right)^2 + \frac{1}{4}\frac{\mu z^4}{a_{\rm h}^4}\right], \qquad n = \frac{\dot{a}}{\dot{a}_{\rm h}},$$

and the inverse relation

$$a_{\rm h}^2 = \frac{a}{2}\left(1+\frac{\mathcal{H}^2 z^2}{2}+\mathcal{E}\sqrt{1+\mathcal{H}^2 z^2-\frac{\mu z^4}{a^4}}\right) \qquad \mathcal{E} = \begin{cases} \pm 1 & \text{one-sided} \\ -1 & \text{two-sided} \end{cases}$$

Holographic map

holographic
cosmology

$$\begin{tikzcd}
z = 0 & \boxed{ds_{\mathrm{h}}^2 = d\tau^2 - a_{\mathrm{h}}^2 d\Omega_k^2} \arrow[r, "\tau \to \tilde{\tau}"] \arrow[d, "z"'] & \boxed{ds_{\mathrm{h}}^2 = \frac{1}{n^2} d\tilde{\tau}^2 - a_{\mathrm{h}}^2 d\Omega_k^2} \arrow[d, "z"] \\
z = z_{\mathrm{br}} & \boxed{ds^2 = n^2 d\tau^2 - a^2 d\Omega_k^2} \arrow[r, "\tau \to \tilde{\tau}"] & \boxed{ds^2 = d\tilde{\tau}^2 - a^2 d\Omega_k^2}
\end{tikzcd}$$

RSII
cosmology

MISCELLANEOUS AND CONCLUSION

- We have investigated a model of inflation based on the dynamics of a D3-brane in the AdS$_5$ bulk of the RSII model. The bulk metric is extended to include the backreaction of the radion excitations.
- The agreement with observations is not ideal, the present model is disfavored but not excluded. However, the model is based on the brane dynamics which results in a definite potential with one free parameter only.
- The simplest tachyon model that stems from the dynamics of a D3-brane in an AdS$_5$ bulk yielding basically an inverse quartic potential.
- The same mechanism lead to a more general tachyon potential if the AdS$_5$ background metric is deformed by the presence of matter in the bulk, e.g. in the form of a minimally coupled scalar field with an arbitrary self-interaction potential. Critical values for the inverse power potential are found.

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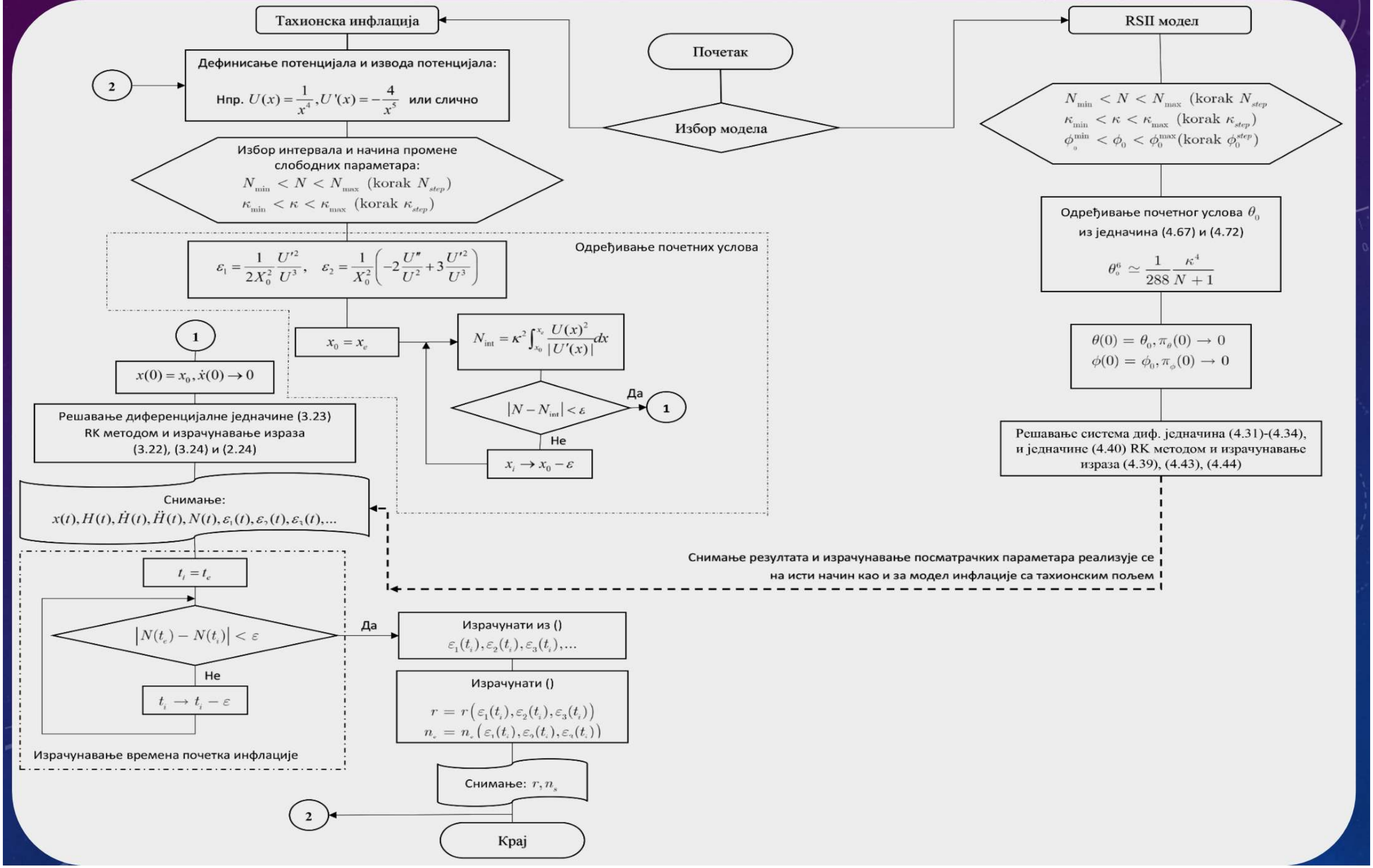
NUMERICAL (PSEUDO)ALGORITAM



Почетак

Избор модела

Тахионска инфлација

Дефинисање потенцијала и извода потенцијала:

Нпр. $U(x)=\frac{1}{x^4}, U'(x)=-\frac{4}{x^5}$ или слично

Избор интервала и начина промене слободних параметара:

$N_{\min} < N < N_{\max}$ (korak N_{step})

$\kappa_{\min} < \kappa < \kappa_{\max}$ (korak κ_{step})

Одређивање почетних услова

$\varepsilon_1=\frac{1}{2X_0^2}\frac{U'^2}{U^3}, \quad \varepsilon_2=\frac{1}{X_0^2}\left(-2\frac{U''}{U^2}+3\frac{U'^2}{U^3}\right)$

$x_0 = x_e$

$N_{\text{int}}=\kappa^2\int_{x_0}^{x_e}\frac{U(x)^2}{|U'(x)|}dx$

$|N-N_{\text{int}}|<\varepsilon$

Да → 1

Не

$x_i \to x_0-\varepsilon$

1

$x(0)=x_0, \dot{x}(0)\to 0$

Решавање диференцијалне једначине (3.23) RK методом и израчунавање израза (3.22), (3.24) и (2.24)

Снимање:

$x(t), H(t), \dot{H}(t), \ddot{H}(t), N(t), \varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t), \ldots$

Израчунавање времена почетка инфлације

$t_i = t_e$

$\left|N(t_e)-N(t_i)\right|<\varepsilon$

Да

Не

$t_i \to t_i-\varepsilon$

Израчунати из ()

$\varepsilon_1(t_i), \varepsilon_2(t_i), \varepsilon_3(t_i), \ldots$

Израчунати ()

$r = r\left(\varepsilon_1(t_i), \varepsilon_2(t_i), \varepsilon_3(t_i)\right)$

$n_s = n_s\left(\varepsilon_1(t_i), \varepsilon_2(t_i), \varepsilon_3(t_i)\right)$

Снимање: r, n_s

2

Крај

RSII модел

$N_{\min} < N < N_{\max}$ (korak N_{step})

$\kappa_{\min} < \kappa < \kappa_{\max}$ (korak κ_{step})

$\phi_0^{\min} < \phi_0 < \phi_0^{\max}$(korak ϕ_0^{step})

Одређивање почетног услова θ_0 из једначина (4.67) и (4.72)

$\theta_0^6 \simeq \frac{1}{288}\frac{\kappa^4}{N+1}$

$\theta(0)=\theta_0, \pi_\theta(0)\to 0$

$\phi(0)=\phi_0, \pi_\phi(0)\to 0$

Решавање система диф. једначина (4.31)-(4.34), и једначине (4.40) RK методом и израчунавање израза (4.39), (4.43), (4.44)

Снимање резултата и израчунавање посматрачких параметара реализује се на исти начин као и за модел инфлације са тахионским пољем

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- Хвала!!!
- …