On inflation in the RSII and holographic braneworld

Goran Djordjević

Department of Physics
Faculty of Sciences and Mathematics
University of Niš & SEENET-MTP CentrE, Serbia

and several collaborators

Gravity and String Theory: New ideas for unsolved problems III
(In honour of Prof. Branislav Sazdović's retirement)
September 7-9 2018, Zlatibor, СРБИЈА

Outline

- Introduction and motivation
- Inflation
- Tachyons
- Braneworld universe and Randall - Sundrum Models (RSI/RSII)
- Numerical results
- Ongoing Research
- Miscellaneous and Conclusion

Introduction and Motivation

Might be inspired by a recent lecture of my colleague Dragan Huterer (Michigan University) during BW2018 in Niš

Could choose 3 key questions in, nowdays, Cosmology

1. Inflation (in Early Universe)
2. Dark matter
3. Dark energy

Introduction and Motivation

1. Inflation (in Early Universe)

- At what energy?
- How many fields?
- With what interactions?

Introduction and Motivation

2. Dark Matter

- What is\are DM particle(s)?
- What are its\their interactions, decay modes … ?

3. Dark Energy

- What is the physics behind the (current) accelerated expansion?

Introduction and Motivation

Background of personal motivation

- Conjectures and papers of Ashoka Sen and others (1999-2002-…)

a) tachyon matter
b) nonarchimedean/*p*-adic mathematical background of strings, branes and tachyons

- *p*-Adic numbers and nonarchimedean geometry in physics (Volovich, Dragovic …)

- *p*-Adic and adelic strings (Volovich, Freund, Witten, Shatashvili, Zwiebach …)
- *p*-Adic inflation (Barnaby, Cline, Koshelev …)
- Extra dimensions and Randal-Sundrum model(s)

Introduction and Motivation

- The inflationary universe scenario in which the early universe undergoes a rapid expansion has been generally accepted as a solution to the horizon problem and some other related problems of the standard big-bang cosmology
- Quantum cosmology: probably the best way to describe the evolution of the early universe, however …
- Recent years - a lot of evidence from WMAP and Planck observations of the CMB

Problems of the Standard Cosmology

- **Horizon:** CMB temperature $T = 2.728 \text{ K}, \ \Delta T / T \sim 10^{-5}$
  Causal horizon at decoupling ct_{dec} subtends $\simeq 1^o$.
- **Flatness:** Friedmann equation: $\Omega - 1 = K / a^2 H^2 \propto a\left(a^2\right)$
  matter (radiation). At e.g. $T = 1 \text{ MeV} \ \Omega - 1 \simeq 10^{-18}$ $m \gg 10^2$
- **Relics:** Extensions of Standard Model contain stable massive particles . E.g. GUT monopoles, SUGRA gravitinos.
- **Fluctuations:** How? Why *10^−5*?
- These features of the Universe are understandable in **inflationary cosmology**.
- **But ...** Big Bang initial singularity?

Inflationary Cosmology

- Inflation means: $\ddot{a} > 0$

- Early Universe had an accelerating phase

- Huge increase in size: “number of e-foldings”

- Quantum fluctuations in a massless scalar field generate perturbations. $N_e \equiv \ln\left(a_{end} / a_i\right) \simeq 60$

Scalar fields in cosmology

$$S = -\int d^4x \sqrt{-g}\,(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi))$$

- Recall FRW metric for flat Universe

$$g_{\mu\nu} = diag(-1, a^2(t))$$

- Field equation: $$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + V'(\phi) = 0$$
- Energy density: $$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\frac{1}{a^2}(\nabla\phi)^2 + V(\phi)$$
- Pressure: $$p = \frac{1}{2}\dot{\phi}^2 + \frac{1}{6}\frac{1}{a^2}(\nabla\phi)^2 - V(\phi)$$

Slow roll inflation

- Postulate that the scalar field is
  - Homogenous: $\phi = \phi(t)$
  - Overdamped: ("slow roll") $\left|\ddot{\phi}\right| \ll 3H\left|\dot{\phi}\right|$
- Sufficient conditions for slow roll:

$$\varepsilon = \frac{1}{2} m_p^2 \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \ll 1, \ \ |\eta| = \left| m_p^2 \frac{V''(\phi)}{V(\phi)} \right| \ll 1$$

- Potential must be "flat" or $\phi \gg m_p$

Slow roll inflation

- Energy density:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) = V(\phi)\left(1 + \frac{1}{3}\varepsilon\right)$$

- Pressure:

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -V(\phi)\left(1 - \frac{1}{3}\varepsilon\right)$$

- Equation of state: $p \simeq -\left(1 - \frac{2}{3}\varepsilon\right)\rho$
- Solution to Friedmann eqn.

$$a(t) \propto t^{1/\varepsilon}\left(\rightarrow \exp(Ht),\ H = \sqrt{V/3m_p^2}\right)$$

Amount of expansion

- Quantified by "number of *e*-foldings" $N_e \equiv \ln\left(a_{end} / a_i\right)$
- Integrate

$$3\frac{\dot{a}}{a}\dot{\phi} = -V'(\phi), \text{ or } \frac{V}{m_p^2}\frac{d\phi}{d\ln a} = -V'(\phi)$$

- Result:

$$N_e = \ln\left(\frac{a_{end}}{a_i}\right) = \frac{1}{m_p^2}\int_{\phi_i}^{\phi_{end}} \frac{V}{V'}d\phi$$

- Example:

$$V = \frac{1}{2}m^2\phi^2 \text{ gives } N_e = \frac{1}{2}\frac{\left(\phi_{end} - \phi_i\right)^2}{m_p^2}$$

End of inflation

- End of inflation: $\varepsilon = 1 \text{ or } |\eta| = 1$
- Example: $V = \frac{1}{2} m^2 \phi^2,\ \varepsilon = 2m_p^2 / \phi^2, \text{ giving } \phi_{end} = \sqrt{2} m_p$
- Field oscillates: $\phi \to \phi_0 \sin(mt)/t,\ a(t) \to t^{2/3}$
- Field decays into other species - (p)reheating
- Thermalisation to energy density $\rho_{rh} < V(\phi_{end})$, temperature T_{rh}
- **NB** T_{rh} must allow nucleosynthesis (*1 MeV*)
- **NB** T_{rh} must allow baryogenesis (*T > 100 GeV*)[a]

[a] *or cold electroweak baryogenesis Smit & Tranberg 2004*

Solving the horizon problem

- Consider mode with comoving momentum *k*, physical inverse wavenumber , compared with Hubble length $\lambda(t) = a(t)/k$ $L_H(t) = H^{-1}$

- Inflation:

$$a(t) \propto t^{1/\varepsilon},\ H^{-1} = \varepsilon t$$

- Radiation:

$$a(t) \propto t^{1/2},\ H^{-1} = 2t$$

- Matter:

$$a(t) \propto t^{2/3},\ H^{-1} = 3t/2$$

Solving the horizon problem

- During inflation the mode's physical wavelength grows faster than the Hubble length. Let $t_1(k)$ be time at which $\lambda(t) = H^{-1}$ during inflation ("horizon exit")

- During standard radiation and matter dominated eras Hubble length grows faster. Let $t_2(k)$ be time at which $\lambda(t) = H^{-1}$ during standard era ("horizon entry").

- **Points that now are not in causal contact were in the same Hubble volume during inflation**

Sufficient inflation

- Modes entering horizon now $\lambda_0(t_0) = a(t_0)/k_0 = H_0^{-1}$
- Require they first crossed horizon (time t_1) during inflation.
- Horizon exit for k_0 mode: $a(t_1)k_0^{-1} = H^{-1}(t_1)$
- Hence $\dfrac{a(t_1)}{a(t_0)} = \dfrac{H_0}{H(t_1)}$
- Assume adiabatic expansion between reheat and today:

$$N_e(t_1) = 67 + \ln\left(\frac{T_{th}}{10^{16}GeV}\right) + \frac{1}{6}\ln\frac{g(T_{rh})}{g(T_0)} + \frac{1}{2}\ln\frac{V(t_1)}{V_{end}} + \frac{1}{2}\ln\frac{V_{end}}{\rho_{rh}} - \frac{1}{3}\ln\frac{a_{rh}}{a_{end}}$$

$$\ln\left(V(t_1)/V_{end}\right) = 2\varepsilon N_e(t_1)$$

- **Require at least about 60 e-folds of inflation**

Solving the flatness problem

- Recall Friedmann equation $\Omega - 1 = K/a^2H^2$
  - Inflation $H^2 \propto a^{-2\varepsilon},\ \Omega(t) - 1 \propto a^{-2(1-\varepsilon)}$
  - Reheating/matter $H^2 \propto a^{-3},\ \Omega - 1 \propto a$
  - Radiation $H^2 \propto a^{-4},\ \Omega - 1 \propto a^2$

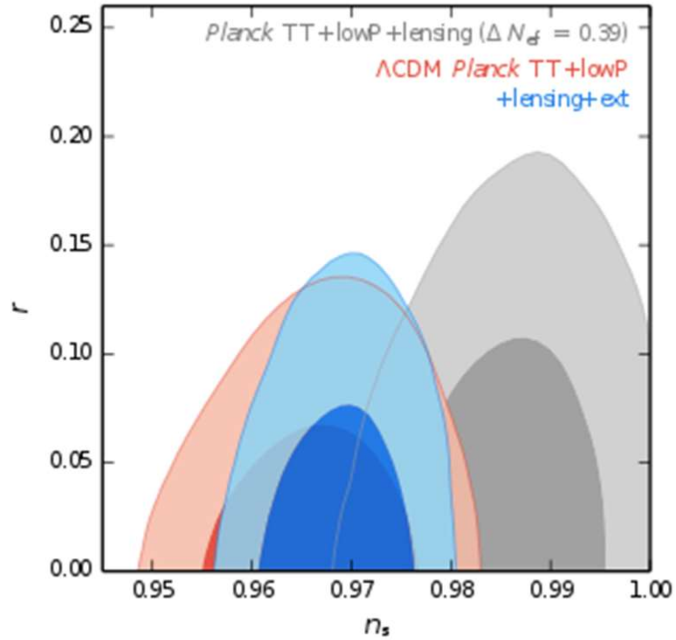
$$\frac{\Omega(t_0) - 1}{\Omega(t_1) - 1} \simeq e^{-2N_e(t_1)} \frac{a_{rh}}{a_{end}} \left(\frac{a_{eq}}{a_{rh}}\right)^2 \frac{a_0}{a_{eq}} \sim e^{-10} \left(\frac{a_{rh}}{a_{end}}\right)^{-4/3}$$

- Have estimated $\frac{\rho_{rh}}{V_{end}} \sim \left(\frac{a_{rh}}{a_{end}}\right)^{-3}$

- **Even if inflation begins at t_1 with $\Omega(t_1) \neq 1$ Universe is now very flat.**

Observational parameters

- Satellite Planck (2009 – 2013 - 2015)
- Latest results are recently published - 2018.



Planck 2015 results: XIII. Cosmological parameters, Astronomy & Astrophysics. 594 (2016) A13
Planck 2015 results. XX. Constraints on inflation, Astronomy & Astrophysics. 594 (2016) A20

Model	Parameter	*Planck* TT+lowP	*Planck* TT+lowP+lensing	*Planck* TT+lowP+BAO	*Planck* TT,TE,EE+lowP
	n_s	0.9666 ± 0.0062	0.9688 ± 0.0061	0.9680 ± 0.0045	0.9652 ± 0.0047
ΛCDM+r	$r_{0.002}$	< 0.103	< 0.114	< 0.113	< 0.099
	n_s	0.9667 ± 0.0066	0.9690 ± 0.0063	0.9673 ± 0.0043	0.9644 ± 0.0049
ΛCDM+r	$r_{0.002}$	< 0.180	< 0.186	< 0.176	< 0.152
$+\mathrm{d}n_s/\mathrm{d}\ln k$	r	< 0.168	< 0.176	< 0.166	< 0.149
	$\mathrm{d}n_s/\mathrm{d}\ln k$	$-0.0126^{+0.0098}_{-0.0087}$	$-0.0076^{+0.0092}_{-0.0080}$	-0.0125 ± 0.0091	-0.0085 ± 0.0076