Entropy and gravitational interaction

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УНИВЕРЗИТЕТ У БЕОГРАДУ
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Gravity and String Theory: New ideas for unsolved problems III,
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Talk overview

Introduction

Black hole entropy in MB model

- Mielke-Baeckler model – action and equations of motion
- BTZ-like black holes with torsion
- "Exotic" black holes with torsion

Black hole entropy in PGT

- 3D gravity with propagating torsion
- Black hole with torsion

Entropy of conformally flat black holes

- Conformally flat Riemannian solutions in PGT
- Static OTT black hole

Conclusion

The talk is based on the following papers

- M. Blagojević and B. Cvetković, Conformally flat black holes in Poincaré gauge theory, PRD **93**, 044018 (2016)
- M. Blagojević, B. Cvetković and M. Vasilić, "Exotic" black holes with torsion, PRD **88**, 101501(R) (2013)
- M. Blagojević and B. Cvetković, 3D gravity with propagating torsion: The AdS sector, PRD **85**, 104003 (2012)
- M. Blagojević and B. Cvetković, Black hole entropy in 3D gravity with torsion, CQG **23**(2006)4781-4795

- In a recently published paper Noether Current, Black Hole Entropy and Spacetime Torsion (arXiv: 1806.0584) authors Sumanta Chakraborty and Ramit Dey claim:
  We show that the presence of spacetime torsion, unlike any other non-trivial modifications of the Einstein gravity, does not affect black hole entropy... We further show that the gravitational Hamiltonian in presence of torsion does not inherit any torsion dependence in the boundary term and hence the first law originating from the variation of the Hamiltonian, relates entropy to area. This reconfirms our claim that torsion does not modify the black hole entropy.
- The authors perform their calculations within Einstein-Cartan theory, where torsion entirely depends on matter contribution. Is this strict conclusion valid within the framework of Poincaré gauge theory (PGT)?

- In his 1993 paper Dirty black holes: Entropy versus area, Phys.Rev. D **48** (1993) 5697-5705, Matt Visser claims: *...On the other hand, the "entropy = (1/4) area" law fails for: various types of (Riemann)n gravity, Lovelock gravity, and various versions of quantum hair. The pattern underlying these results is less than clear.*
- Our final goal is to examine the deviation from the Bekenstein-Hawking area law $S = \frac{A}{4G}$ for the various black hole solution within the framework of 4D PGT.
- In that we shall be able to examine the influence that both torsional and curvature terms have on black hole entropy.
- In this talk we review results for the entropy of black holes which are the exact solutions of the various 3D gravity models in the framework of PGT.

- The common feature is the devitation of the black hole entropy from the Bekenstein-Hawking area law $S = \frac{A}{4G}$ due to the presence of the non-Einstein terms in the gravitational Lagrangian.
- In some cases entropy depends explicitly on torsion, and there is a deviation from the Bekenstein-Hawking area law even for Riemannian solutions of PGT.
- It is worth noting that results obtained within PGT formalism can be reduced to the ones obtained within TMG and BHT gravity.
- The first law of black hole thermodynamics is satisfied in all the cases considered.

- In the framework of Poincaré gauge theory, the triad fields b^i and the Lorentz connection ω^{ij} are basic dynamical variables (1-forms).
- Their field strengths, expressed in terms of the Lie dual connection $\omega^i := -\frac{1}{2}\varepsilon^{ijk}\omega_{jk}$ are the torsion $T^i = db^i + \varepsilon^{ijk}\omega_j b_k$ and the curvature $R^i = d\omega^i + \frac{1}{2}\varepsilon^{ijk}\omega_j\omega_k$ (2-forms).
- In this framework the MB model is defined by the Lagrangian (3-form)

$$L_{\rm MB} = 2ab^i R_i - \frac{\Lambda}{3}\varepsilon_{ijk}b^i b^j b^k + \alpha_3 L_{\rm CS}(\omega) + \alpha_4 b^i T_i\,. \quad (2.1)$$

- Here, $L_{\rm CS}(\omega) := \omega^i d\omega_i + \frac{1}{3}\varepsilon_{ijk}\omega^i\omega^j\omega^k$ is the Chern–Simons Lagrangian for ω^i, the exterior product is omitted for simplicity, and $(a, \Lambda, \alpha_3, \alpha_4)$ are free parameters.

- In the non-degenerate case $\alpha_3\alpha_4 - a^2 \neq 0$, the variation of $L_{\rm MB}$ with respect to b^i and ω^i leads to the gravitational field equations in vacuum:

$$2T^i = p\varepsilon^i{}_{jk}\, b^j b^k\,, \qquad 2R^i = q\varepsilon^i{}_{jk}\, b^j b^k\,, \qquad (2.2)$$

$$p = \frac{\alpha_3\Lambda + \alpha_4 a}{\alpha_3\alpha_4 - a^2}\,, \qquad q = -\frac{(\alpha_4)^2 + a\Lambda}{\alpha_3\alpha_4 - a^2}\,. \qquad (2.3)$$

- By using Eqs. (2.2) and the formula $\omega^i = \tilde\omega^i + K^i$, where $\tilde\omega^i$ is the Riemannian (torsionless) connection, and K^i is the contortion 1-form, defined implicitly by $T_i = \varepsilon_{imn}K^m e^n$, one can show that the Riemannian piece of the curvature is:

$$2\tilde R^i = \Lambda_{\rm eff}\,\varepsilon^i{}_{jk} e^j e^k\,, \qquad \Lambda_{\rm eff} := q - \frac{1}{4}p^2\,, \qquad (2.4)$$

where $\Lambda_{\rm eff}$ is the effective cosmological constant.

- In the AdS sector with $\Lambda_{\rm eff} = -1/\ell^2$, the MB model admits a new type of black hole solutions, the BTZ-like *black holes with torsion*. From the form the BTZ black hole metric

$$ds^2 = N^2 dt^2 - N^{-2} dr^2 - r^2 (d\varphi + N_\varphi dt)^2 ,$$
$$N^2 = \left(-8Gm + \frac{r^2}{\ell^2} + \frac{16G^2 j^2}{r^2} \right) , \quad N_\varphi = \frac{4Gj}{r^2} ,$$

and the relation $ds^2 = \eta_{ij} b^i b^j$, one concludes that the triad field can be chosen in the simple, diagonal form:

$$b^0 = N dt , \quad b^1 = N^{-1} dr , \quad b^2 = r\,(d\varphi + N_\varphi dt) . \quad (2.5a)$$

- The connection is determined by:

$$\omega^i = \tilde{\omega}^i + \frac{p}{2} e^i . \quad (2.5b)$$

- BTZ-like black hole with torsion is represented by (b^i, ω^i).

- Energy and angular momentum of the black hole with torsion, defined as the on-shell values of the asymptotic generators for time translations and spatial rotations are:

$$E = 16\pi G\left[\left(a+\frac{\alpha_3 p}{2}\right)m - \frac{\alpha_3}{\ell^2}j\right] ,$$
$$J = 16\pi G\left[\left(a+\frac{\alpha_3 p}{2}\right)j - \alpha_3 m\right] . \qquad (2.6)$$

- In contrast to GR_Λ, where $E = m$ and $J = j$, the presence of the Chern–Simons term ($\alpha_3 \neq 0$) modifies E and J into linear combinations of m and j.
- After choosing the AdS asymptotic conditions, the PB algebra of the *asymptotic symmetry* is given by two Virasoro algebras with different central charges:

$$c^{\mp} = 24\pi\left[\left(a+\frac{\alpha_3 p}{2}\right)\ell \mp \alpha_3\right] . \qquad (2.7)$$

- The partition function of the MB model, calculated in the semiclassical approximation around the black hole with torsion, yields the following expression for the *black hole entropy*:

$$S = 8\pi^2 \left[\left(a + \frac{\alpha_3 p}{2} \right) r_+ - \alpha_3 \frac{r_-}{\ell} \right] , \qquad (2.8)$$

  where $r_\pm$ are the outer and inner horizons of the black hole, defined as the zeros of N^2.
- The entropy differs from Bekenstein-Hawking result by an additional term, which describes the torsional degrees of freedom at the outer horizon and degrees of freedom at the inner horizon.
- The result for black hole entropy in the absence of torsion $p = 0$ coincides with the result for the BTZ black hole entropy in TMG obtained by Solodukhin.

- The gravitational entropy coincides with the corresponding statistical or *conformal* entropy, obtained by combining Cardy's formula with the central charges (2.7):

$$S = 2\pi\sqrt{\frac{h^- c^-}{6}} + 2\pi\sqrt{\frac{h^+ c^+}{6}}\,, \qquad (2.9)$$

where $h^\mp = \frac{1}{2}(\ell E \pm J)$.

- The existence of torsion is shown to be in *complete agreement* with the first law of black hole thermodynamics:

$$T\delta S = \delta E - \Omega\delta J\,, \qquad (2.10)$$

where

$$T = \frac{1}{4\pi}\partial_r N^2|_{r=r_+} = \frac{r_+^2 - r_-^2}{2\pi\ell^2 r_+^2}\,, \qquad \Omega = N_\varphi(r_+) = \frac{r_-}{\ell r_+}\,,$$

are black hole temperature and angular velocity.

- The two types of black holes discussed Townsend and Zhang can be given a unified treatment by considering the related limiting cases of the MB model.
- For $\alpha_3 = \alpha_4 = 0$ and $16\pi Ga = 1$, the MB model reduces to GR_Λ, the spacetime geometry is Riemannian ($p = 0$), and

$$E = m, \quad J = j, \quad c^\mp = \frac{3\ell}{2G}, \quad S = \frac{2\pi r_+}{4G}. \qquad (2.11)$$

- For $a = \Lambda = 0$, the MB model reduces to Witten's “exotic" gravity with Riemannian geometry of spacetime. By choosing $16\pi G\alpha_3 = -\ell$, one arrives at the “exotic" conserved charges, central charges and entropy,

$$E = \frac{j}{\ell}, \quad J = \ell m, \quad c^\mp = \pm\frac{3\ell}{2G}, \quad S = \frac{2\pi r_-}{4G}, \qquad (2.12)$$

which coincide with Townsend's and Zhang's ones.

- The concepts of standard and “exotic" black holes used in the context of simple gravitational models with Riemannian geometry of spacetime can be generalized by going over to black holes with torsion.
- The form of the general results (2.6), (2.7) and (2.8) suggests to introduce *standard* black holes with torsion by imposing the following requirements:

$$\alpha_3 = 0\,, \qquad 16\pi G a = 1\,. \tag{2.13}$$

- In this case, the general formulas reduce to the standard form (2.11), and the corresponding 2-parameter Lagrangian is given by:

$$L_S = \frac{1}{8\pi G} b^i R_i - \frac{\Lambda}{3}\varepsilon_{ijk} b^i b^j b^k + \alpha_4 b^i T_i\,. \tag{2.14}$$

- Definition of *"exotic"* black holes with torsion:

$$a+\frac{\alpha_3 p}{2}=0\,,\qquad 16\pi G\alpha_3=-\ell\,, \qquad (2.15)$$

  implies that the conserved charges, central charges and entropy take the "exotic" form (2.12).
- The corresponding 2-parameter Lagrangian is

$$L_{\mathrm{E}} = \frac{1}{16\pi G}\left[2\beta b^i R_i+\frac{\beta(\beta^2+3)}{3\ell^2}\varepsilon_{ijk}b^i b^j b^k -\ell L_{CS}-\frac{\beta^2+1}{\ell}b^i T_i\right]\,, \qquad (2.16)$$

  where $\beta := 16\pi Ga$ and ℓ are free parameters.
- In the limit $p=0$, L_{S} and L_{E} describe torsionless theories discussed by Townsend and Zhang. All the other limits define the standard and "exotic" gravities *with torsion*.

- MB model is introduced as a *topological* 3D gravity with torsion, with an idea to explore the influence of geometry on the dynamics of gravity.
- GR_Λ in 3D is also a topological theory, which has no propagating degrees of freedom.
- Such a degenerate situation is not quite a realistic feature of the gravitational dynamics and one is naturally motivated to study gravitational models *with propagating degrees of freedom*.
- Within Riemannian geometry, there are two well-known models of this type: TMG and the BHT massive gravity.
- In 3D gravity with torsion, an extension that includes propagating modes is even more natural—it corresponds to Lagrangians which are *quadratic* in the field strengths, as in the standard gauge approach.

- General dynamics of 3D gravity with propagating torsion is defined by the Lagrangian 3-form

$$L = L_G(b^i, T^i, R^{ij}) + L_M(b^i, \psi, \nabla\psi) \tag{3.1a}$$

where L_M denotes matter contribution, and the gravitational piece L_G is at most quadratic in torsion and curvature. Assuming that L_G preserves parity, we have

$$\begin{aligned}
L_G &= -a\varepsilon_{ijk} b^i \wedge R^{jk} - \frac{1}{3}\Lambda_0 \varepsilon_{ijk} b^i \wedge b^j \wedge b^k + L_{T^2} + L_{R^2}\,, \\
L_{T^2} &= T^i \wedge {}^\star \left(a_1 {}^{(1)}T_i + a_2 {}^{(2)}T_i + a_3 {}^{(3)}T_i \right)\,, \\
L_{R^2} &= \frac{1}{2} R^{ij} \wedge {}^\star \left(b_4 {}^{(4)}R_{ij} + b_5 {}^{(5)}R_{ij} + b_6 {}^{(6)}R_{ij} \right)\,,
\end{aligned} \tag{3.1b}$$

where ${}^{(a)}T_i$ and ${}^{(a)}R_{ij}$ are irreducible components of the torsion and the RC curvature.

- The covariant gravitational momenta (1-forms) are

$$H_i := \frac{\partial L_G}{\partial T^i}\,, \qquad H_{ij} := \frac{\partial L_G}{\partial R^{ij}}\,. \tag{3.2}$$

- Dynamical energy-momentum and spin currents (2-forms) for the gravitational field and matter currents (2-forms) are:

$$\begin{aligned} t_i &:= \frac{\partial L_G}{\partial b^i}\,, \qquad & s_{ij} &:= \frac{\partial L_G}{\partial A^{ij}}\,, \\ \tau_i &:= \frac{\partial L_M}{\partial b^i}\,, \qquad & \sigma_{ij} &:= \frac{\partial L_M}{\partial A^{ij}} = \Sigma_{ij}\psi\, \frac{\partial L_M}{\partial \nabla\psi}\,. \end{aligned} \tag{3.3}$$

- The variation of the Lagrangian (3.1a) with respect to b^i and A^{ij} produces the following gravitational field equations:

$$\nabla H_i + t_i = -\tau_i\,, \tag{3.4a}$$

$$\nabla H_{ij} + s_{ij} = -\sigma_{ij}\,. \tag{3.4b}$$

- Explicit calculation based on the gravitational Lagrangian (3.1b) yields

$$H_i = 2^\star\left(a_1{}^{(1)}T_i + a_2{}^{(2)}T_i + a_3{}^{(3)}T_i\right)\,,$$

$$H_{ij} = -2a\varepsilon_{ijk}b^k + H'_{ij}\,,\qquad H'_{ij} := 2^\star\left(b_4{}^{(4)}R_{ij} + b_5{}^{(5)}R_{ij} + b_6{}^{(6)}R$$

and

$$\begin{aligned} t_i &= e_i \rfloor L_G - (e_i \rfloor T^m)\wedge H_m - \frac{1}{2}(e_i \rfloor R^{mn})\wedge H_{mn}\,,\\ s_{ij} &= -\left(b_i\wedge H_j - b_j\wedge H_i\right)\,. \end{aligned} \qquad (3.6)$$

- The gravitational Lagrangian can be written in a more compact form as:

$$L = \frac{1}{2}T^iH_i + \frac{1}{2}R^{ij}(-2a\varepsilon_{ijk}b^k) + \frac{1}{4}R^{ij}H'_{ij} - \frac{1}{3}\Lambda_0\varepsilon_{ijk}b^ib^jb^k\,.$$

- 3D gravity with propagating torsion admits the existence of AdS black hole solution, BTZ-like black hole with torsion of the MB model.
- Let us recall that field strenghts have the form:

$$2T_i = p\varepsilon_{ijk}b^j b^k\,, \qquad 2R_i = q\varepsilon_{ijk}b^j b^k\,, \tag{3.7}$$

where p and q are parameters, and we assume that the effective cosmological constant is negative.
- By combining (3.7) with the field equations in vacuum, we can obtain restrictions on p and q, under which the BTZ-like black hole is an exact solution of the theory:

$$\begin{aligned} &aq - \Lambda_0 + \frac{1}{2}p^2 a_3 - \frac{1}{2}q^2 b_6 = 0\,, \\ &p(a + qb_6 + 2a_3) = 0\,. \end{aligned} \tag{3.8}$$

- These conditions guarantee that the black hole with torsion is a solution of the PGT model (4.1). The second equation naturally leads to the following two cases:

  a) $p = 0 \quad \Rightarrow$
  For $b_6 \neq 0$, we have

  $$qb_6 = a \pm \sqrt{a^2 - 2b_6\Lambda_0}\,.$$

  If, additionally, $a^2 - 2b_6\Lambda_0 = 0$, the value of qb_6 is unique: $qb_6 = a$.
  For $b_6 = 0$, the value of q is $q = \Lambda_0/a$.

  b) $a + qb_6 + 2a_3 = 0 \quad \Rightarrow$

  $$\frac{1}{2}a_3p^2 = \Lambda_0 + \frac{1}{2}q(qb_6 - 2a) = \Lambda_0 + \frac{1}{2b_6}(2a_3 + a)(2a_3 + 3a)\,.$$

  For $a_3 = 0$, p remains undetermined, which is physically not acceptable.

- Energy and angular momentum of the black hole with torsion are:

$$E=\left(1+\frac{qb_6}{a}\right)m\,,\qquad J=\left(1+\frac{qb_6}{a}\right)j\,. \quad (3.9)$$

- The conserved charges depend on the curvature strength q but not on the torsion strength p. For $qb_6\neq 0$, the values of the black hole charges differ from the corresponding GR expressions.
- After choosing the AdS asymptotic conditions, the Poisson bracket algebra of the *asymptotic symmetry* is given by two independent Virasoro algebras with equal central charges:

$$c^-=c^+=\left(1+\frac{qb_6}{a}\right)\frac{3\ell}{2G}\,. \quad (3.10)$$

- Once we have the central charges, we can use Cardy's formula to calculate the black hole entropy:

$$S = \left(1 + \frac{qb_6}{a}\right) \frac{2\pi r_+}{4G}\,, \tag{3.11}$$

where r_+ is the radius of the outer black hole horizon.

- With the above results for the conserved charges and entropy, one can easily verify the validity of the first law of black hole thermodynamics.
- The similar result for the entropy of the BTZ black holes holds in BHT gravity.

Conformally flat Riemannian solutions in PGT

- The OTT black hole is a vacuum solution of the BHT gravity with a unique AdS ground state. It is also a Riemannian solution of PGT in vacuum due to a deep dynamical relation between the Riemannian sector of PGT and the BHT gravity.
- The content of this relation is expressed by a theorem stating that any *conformally flat* solution of the BHT gravity is also a Riemannian solution of PGT.
- This is, in particular, true for the OTT black holes.
- In 3D, the Weyl curvature identically vanishes, and the Cotton 2-form C^i is used to characterize conformal properties of spacetime.
- It is defined by $C^i := \nabla L^i = dL^i + \omega^i{}_m L^m$ where $L^m := Ric^m - \frac{1}{4} R b^m$ is the Schouten 1-form. A spacetime is conformally flat when $C^i = 0$.

- The BHT gravity action

$$I_{\rm BHT} = a_0 \int d^3x\sqrt{g}\left(R - \lambda + \frac{1}{m^2}K\right)\,, \qquad K := Ric^{ij}Ric_{ij} - \frac{3}{8}R^2\,,$$

leads to the field equations:

$$G_{ij} - \lambda\eta_{ij} - \frac{1}{2m^2}K_{ij} = 0\,, \tag{4.1}$$
$$K_{ij} = K\eta_{ij} - 2L_{ik}G^k{}_j - 2(\nabla_m C_{in})\varepsilon^{mn}{}_j\,,$$

- In PGT, the gravitational Lagrangian is at most quadratic in the torsion T^i and the curvature R^{ij}.
- A Riemannian curvature in 3D has only two nonvanishing irreducible components,

$$^{(6)}R^{ij} = \frac{1}{6}Rb^ib^j\,, \qquad ^{(4)}R_{ij} = R^{ij} - {}^{(6)}R^{ij}\,.$$

- For quadratic and parity-invariant L_G, the Riemannian reduction of the general field equations takes the form:

$$
\begin{aligned}
&(1\mathrm{ST}) \qquad E_i = 0\,, \\
&(2\mathrm{ND}) \qquad \nabla H_{ij} = 0\,, \qquad (4.2\mathrm{a})
\end{aligned}
$$

$$
E_i = h_i \rfloor L_G - \frac{1}{2}(h_i \rfloor R^{mn})H_{mn}\,,
$$

$$
H_{ij} = -2a_0\varepsilon_{ijm}b^m + \frac{b_4 + 2b_6}{6}R\varepsilon_{ijk}b^k - 2b_4\varepsilon_{ij}{}^m L_m\,. \qquad (4.2\mathrm{b})
$$

- Let us now note a simple property of (2ND): the vanishing of the second term in H_{ij} implies that the Cotton 2-form $C_m = \nabla L_m$ vanishes. More precisely:

(T1) A Riemannian solution of PGT is conformally flat iff $b_4 + 2b_6 = 0$.

- Next, to examine the content of (1ST), it is convenient to express it in the form:

$$a_0 G_{ij} - \Lambda_0 \eta_{ij} - b_4 \frac{1}{2} \left(K\eta_{ij} - 2L_{im} G^m{}_j \right) = 0 \,. \qquad (4.3)$$

- A direct comparison shows that Eq. (4.3) coincides with the BHT field equation (4.1) for $C_{in} = 0$, provided one makes the following identification of parameters:

$$\Lambda_0 = a_0 \lambda \,, \qquad b_4 = a_0/m^2 \,. \qquad (4.4)$$

This leads to the result:

(T2) Any conformally flat solution of the BHT gravity is also a Riemannian solution of PGT with $b_4 + 2b_6 = 0$, and vice versa.

Static OTT black hole

- The static OTT spacetime is described by the metric

$$ds^2 = N^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\varphi^2\,, \qquad N^2 := -\mu + br + \frac{r^2}{\ell^2}\,, \quad (4.5)$$

where μ and b are real parameters. The roots of equation $N^2 = 0$ are

$$r_\pm = \frac{1}{2}\left(-b\ell^2 \pm \ell\sqrt{4\mu + b^2\ell^2}\right)\,.$$

- For $b = 0$ it reduces to the BTZ black hole.
- The triad field reads

$$b^0 := Ndt\,, \qquad b^1 := \frac{dr}{N}\,, \qquad b^2 := rd\varphi\,, \qquad (4.6a)$$

while the corresponding Riemannian connection is:

$$\omega^{01} = -(\partial_r N)b^0\,, \qquad \omega^{02} = 0\,, \qquad \omega^{12} = \frac{N}{r}b^2\,. \quad (4.6b)$$

- The geometric structure introduced in Eqs. (4.6) can be now used to calculate first the curvature 2-form R^{ij}, and then the Schouten 1-form.
- An explicit calculation yields $C^i = \nabla L^i = 0$, and theorem (T2) implies that the static OTT black hole is an exact Riemannian solution of PGT in vacuum.
- The values of the improved generators for time translations are given by the corresponding boundary terms, which define the conserved charges of the system, the energy and the angular momentum, respectively:

$$E = \frac{1}{4G}\left(\mu + \frac{1}{4}b^2\ell^2\right), \qquad J = 0\,. \tag{4.7}$$

- The black hole entropy can be calculated from the Cardy formula:

$$S = 2\pi\ell\sqrt{\frac{E}{G}} = \frac{2\pi(r_+ + r_-)}{2G}\,. \tag{4.8}$$

- Using the expression for the Hawking temperature,

$$T = \frac{1}{4\pi}\,\partial_r N^2\Big|_{r=r_+} = \frac{1}{\pi\ell}\sqrt{GE}\,, \tag{4.9}$$

  one can directly verify the first law of the black hole thermodynamics:

$$\delta E = T\delta S\,. \tag{4.10}$$

- Since the entropy vanishes for $E = 0$, the state with $E = 0$ can be naturally regarded as the ground state of the OTT family of black holes.

- We reviewed results for the black hole entropy for BTZ black hole with torsion within MB model and 3D gravity with propagating torsion as well as OTT black hole within 3D PGT.
- Entropy is (not necessarily) influenced by the spacetime torsion and deviates from the Bekenstein-Hawking area law.
- All the results are in accordance first law of black hole thermodynamics.